

講義中の注意



- 講義中は、参加者のマイク・カメラの機能はミュート状態になります。
- 進行はスタッフ及び講師が行いますので、指示に従ってください。
- 質疑応答の時間は、参加者のマイクをオンにして質問を受け付けることもあります。希望される方は「チャット欄」で申し出てください。

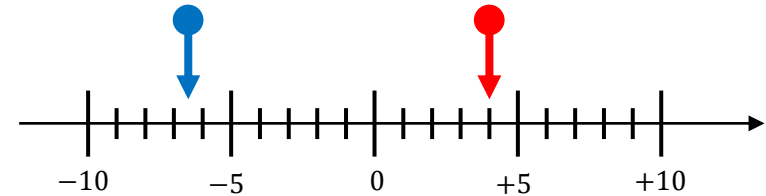
電験三種 ライブ講義

第8回 ベクトル

ベクトル

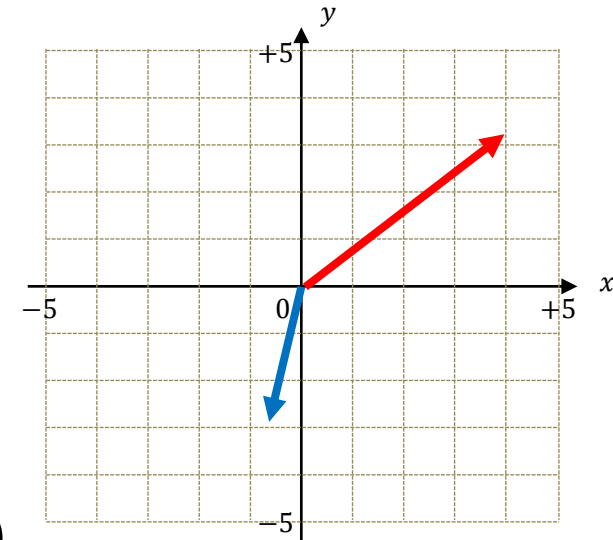
スカラー量 “大きさ”のみ

例) 時間、重さ、温度、面積、エネルギーなど
電荷、静電容量、電力など



ベクトル量 “大きさ”と“向き”

例) 位置、速度、加速度、力など
電流、電圧、インピーダンス、電界、磁界など



ベクトルの表し方

\vec{a}, \vec{b} 高校数学での表現

a, b 大学や専門科目での表現 (電磁気学)

\dot{a}, \dot{b} ベクトル (複素平面) の表現 ← 電験はこれ

ベクトルを理解するために



計算に必要な知識

- A. xy 平面の座標の読み方
- B. 三平方の定理
- C. 三角関数

ベクトルとして知っておくこと

- 1. 位置ベクトル
- 2. ベクトルの大きさ
- 3. ベクトルの成分分解
- 4. ベクトルの合成

位置ベクトル

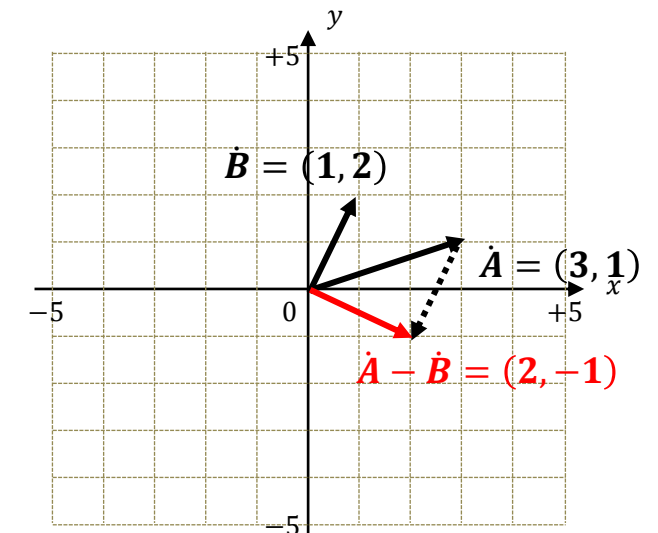
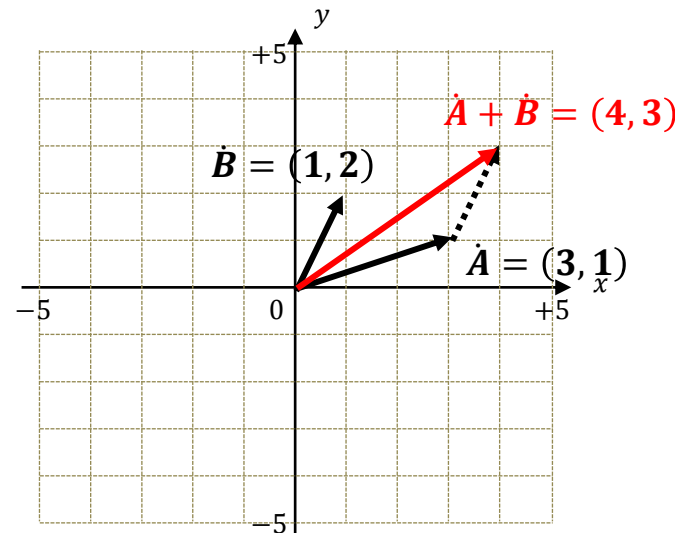
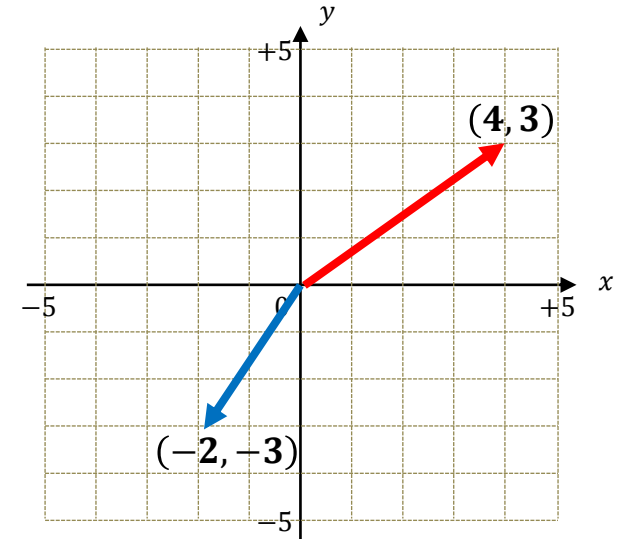
位置ベクトル: xy 平面上で原点 O からの位置 (x, y) を表すベクトル

位置ベクトル同士の足し算、引き算

$$\dot{A} = (a, b), \dot{B} = (c, d)$$

$$\dot{A} + \dot{B} = (a + c, b + d)$$

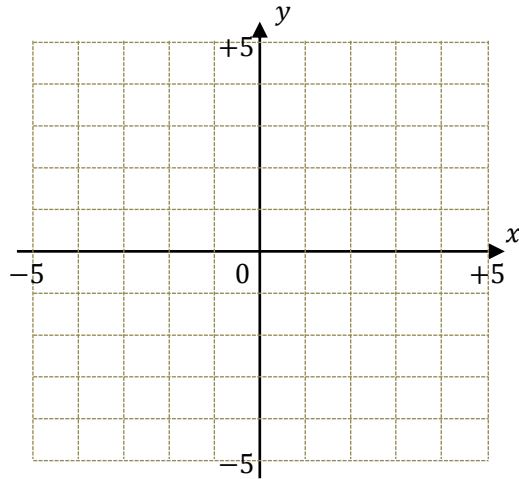
$$\dot{A} - \dot{B} = (a - c, b - d)$$



練習問題 I

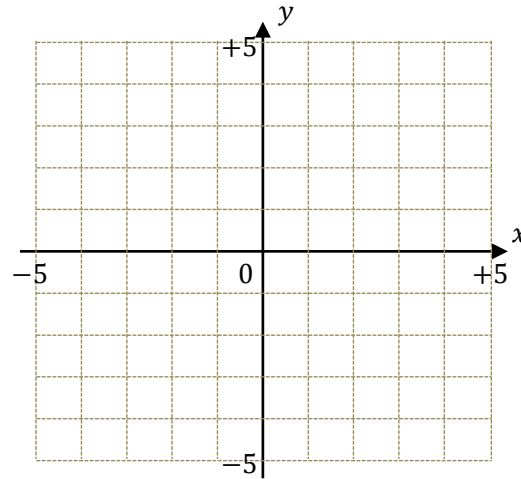
(1)

$$\begin{aligned}\dot{A} &= (1, 0) \\ \dot{B} &= (2, 0) \\ \dot{C} &= \dot{A} + \dot{B}\end{aligned}$$



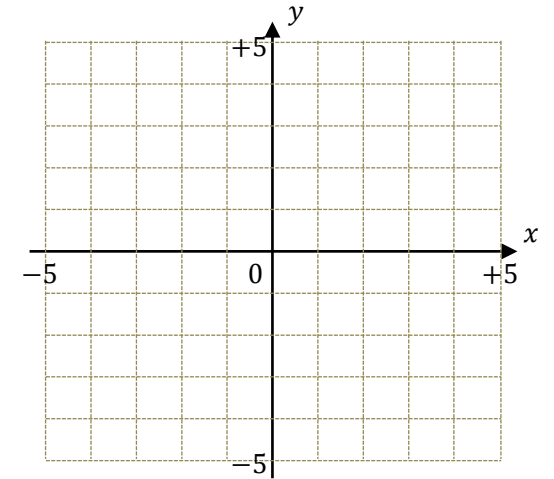
(2)

$$\begin{aligned}\dot{A} &= (2, 1) \\ \dot{B} &= (1, 2) \\ \dot{C} &= \dot{A} + \dot{B}\end{aligned}$$



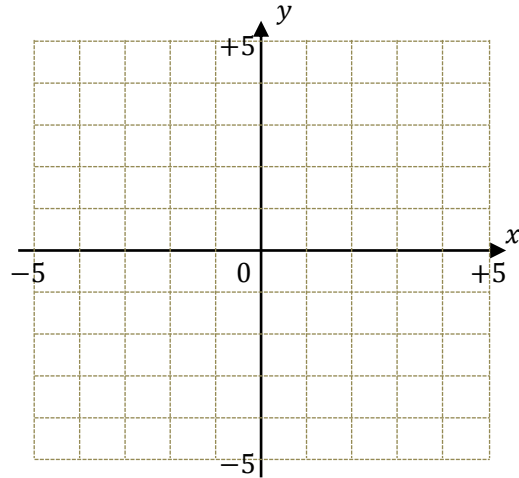
(3)

$$\begin{aligned}\dot{A} &= (4, 1) \\ \dot{B} &= (-2, 2) \\ \dot{C} &= \dot{A} + \dot{B}\end{aligned}$$



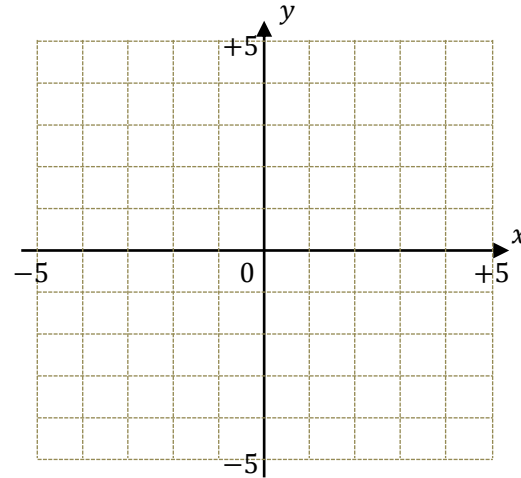
(4)

$$\begin{aligned}\dot{A} &= (-1, -1) \\ \dot{B} &= (-2, -3) \\ \dot{C} &= \dot{A} + \dot{B}\end{aligned}$$



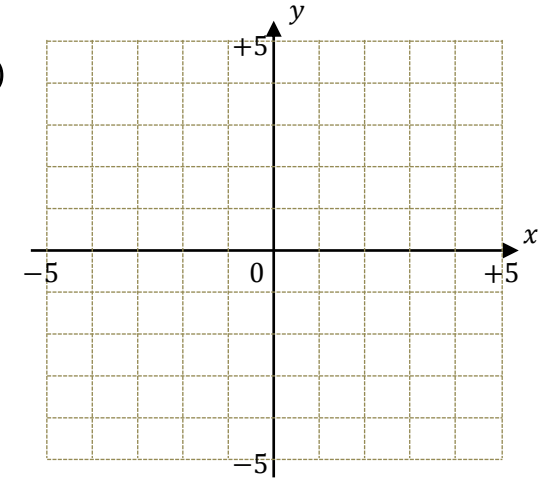
(5)

$$\begin{aligned}\dot{A} &= (5, 2) \\ \dot{B} &= (2, 2) \\ \dot{C} &= \dot{A} - \dot{B}\end{aligned}$$



(6)

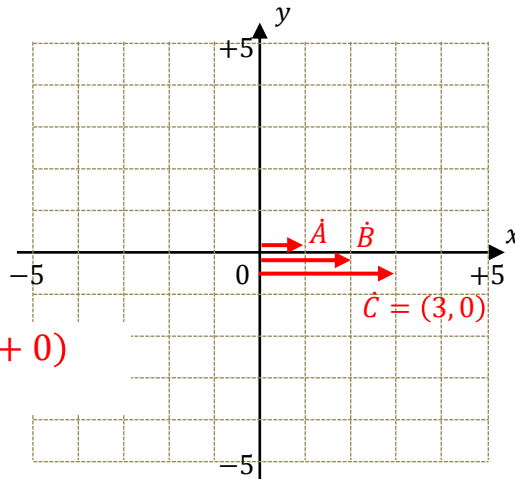
$$\begin{aligned}\dot{A} &= (1, 2) \\ \dot{B} &= (-2, -2) \\ \dot{C} &= \dot{A} - \dot{B}\end{aligned}$$



練習問題 I

(1)

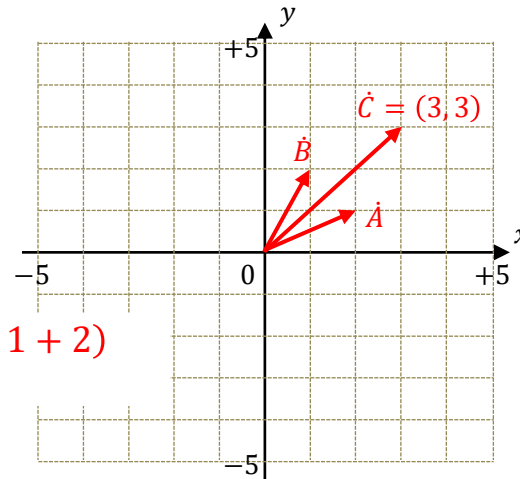
$$\begin{aligned} \vec{A} &= (1, 0) \\ \vec{B} &= (2, 0) \\ \vec{C} &= \vec{A} + \vec{B} \end{aligned}$$



$$\begin{aligned} \vec{C} &= (2 + 1, 0 + 0) \\ &= (3, 0) \end{aligned}$$

(2)

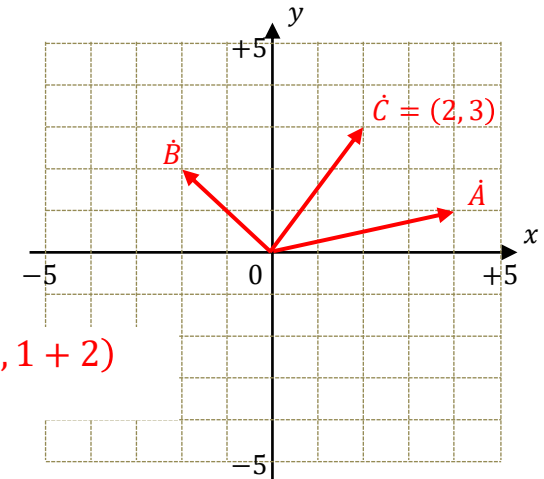
$$\begin{aligned} \vec{A} &= (2, 1) \\ \vec{B} &= (1, 2) \\ \vec{C} &= \vec{A} + \vec{B} \end{aligned}$$



$$\begin{aligned} \vec{C} &= (2 + 1, 1 + 2) \\ &= (3, 3) \end{aligned}$$

(3)

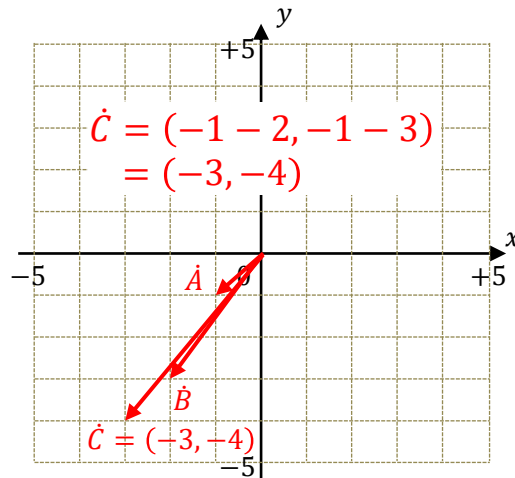
$$\begin{aligned} \vec{A} &= (4, 1) \\ \vec{B} &= (-2, 2) \\ \vec{C} &= \vec{A} + \vec{B} \end{aligned}$$



$$\begin{aligned} \vec{C} &= (4 - 2, 1 + 2) \\ &= (2, 3) \end{aligned}$$

(4)

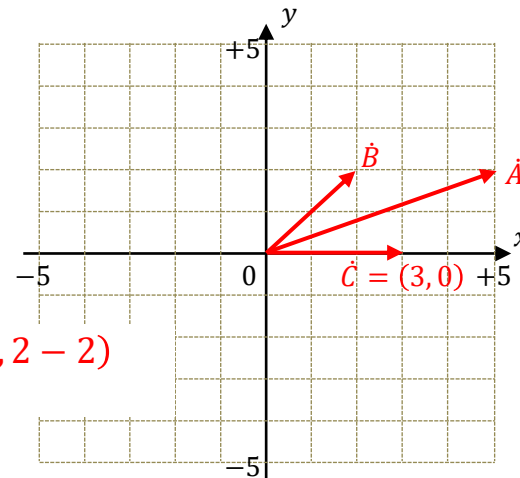
$$\begin{aligned} \vec{A} &= (-1, -1) \\ \vec{B} &= (-2, -3) \\ \vec{C} &= \vec{A} + \vec{B} \end{aligned}$$



$$\begin{aligned} \vec{C} &= (-1 - 2, -1 - 3) \\ &= (-3, -4) \end{aligned}$$

(5)

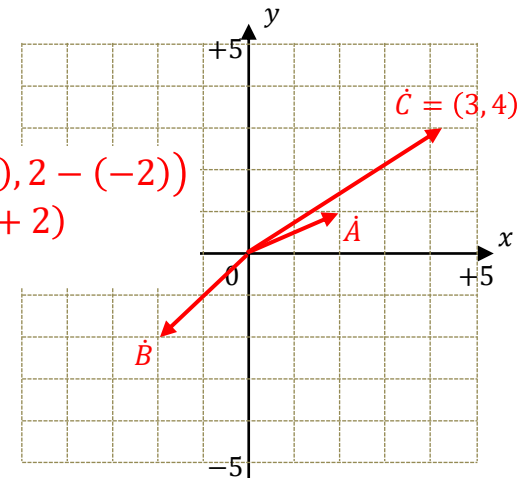
$$\begin{aligned} \vec{A} &= (5, 2) \\ \vec{B} &= (2, 2) \\ \vec{C} &= \vec{A} - \vec{B} \end{aligned}$$



$$\begin{aligned} \vec{C} &= (5 - 2, 2 - 2) \\ &= (3, 0) \end{aligned}$$

(6)

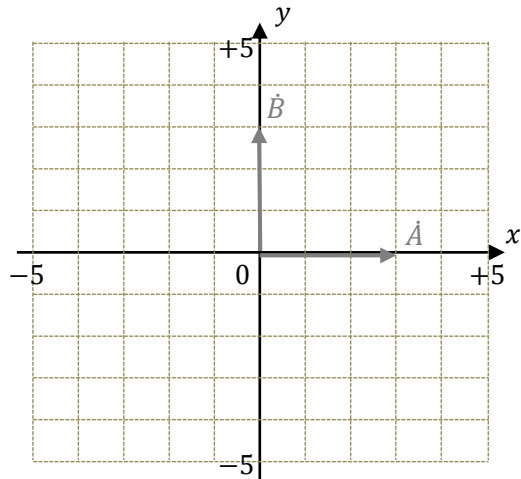
$$\begin{aligned} \vec{A} &= (1, 2) \\ \vec{B} &= (-2, -2) \\ \vec{C} &= \vec{A} - \vec{B} \end{aligned}$$



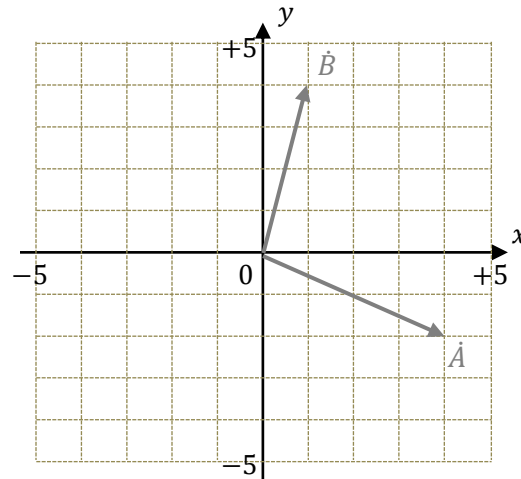
$$\begin{aligned} \vec{C} &= (1 - (-2), 2 - (-2)) \\ &= (1 + 2, 2 + 2) \\ &= (3, 4) \end{aligned}$$

練習問題2

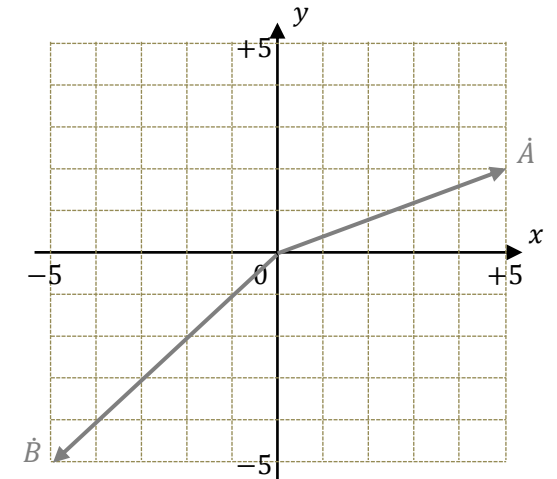
(1)
 $\vec{C} = \vec{A} + \vec{B}$



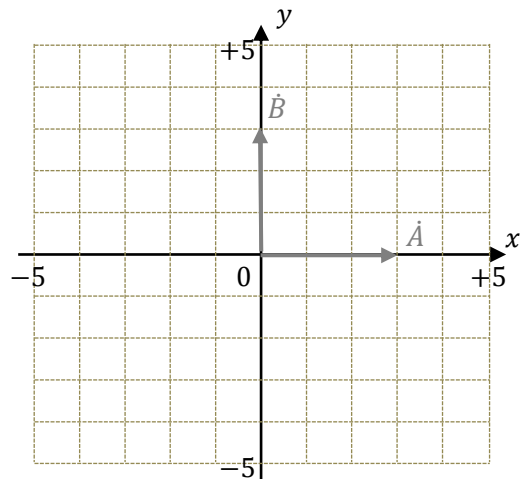
(2)
 $\vec{C} = \vec{A} + \vec{B}$



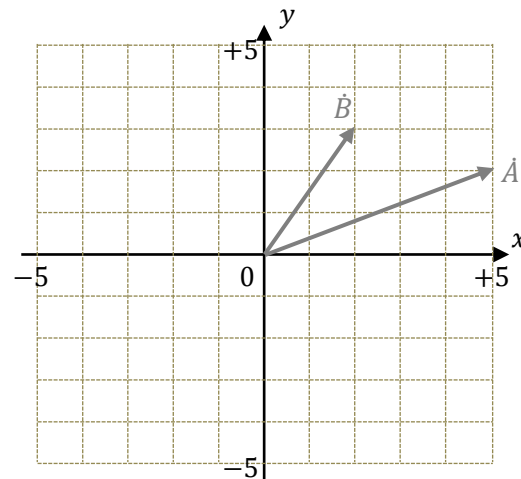
(3)
 $\vec{C} = \vec{A} + \vec{B}$



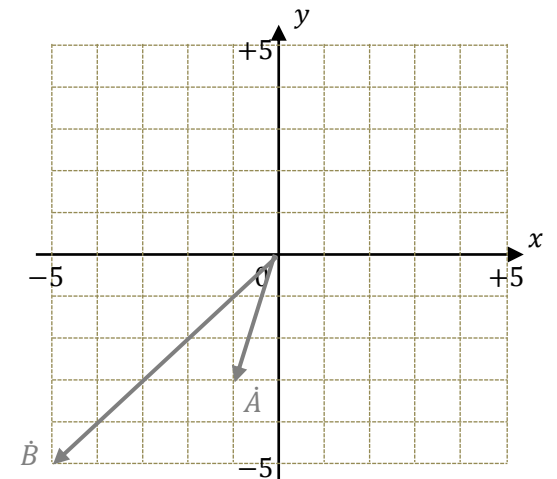
(4)
 $\vec{C} = \vec{A} - \vec{B}$



(5)
 $\vec{C} = \vec{A} - \vec{B}$

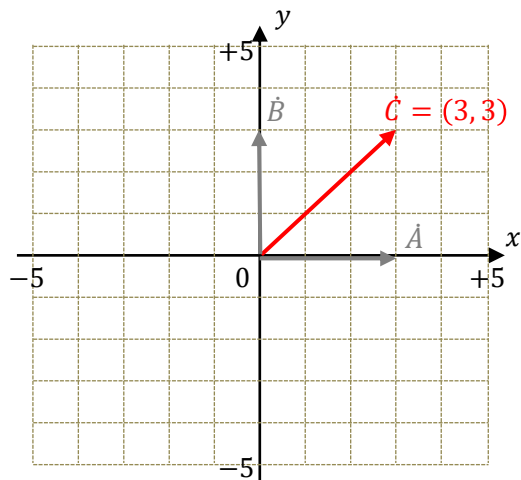


(6)
 $\vec{C} = \vec{A} - \vec{B}$

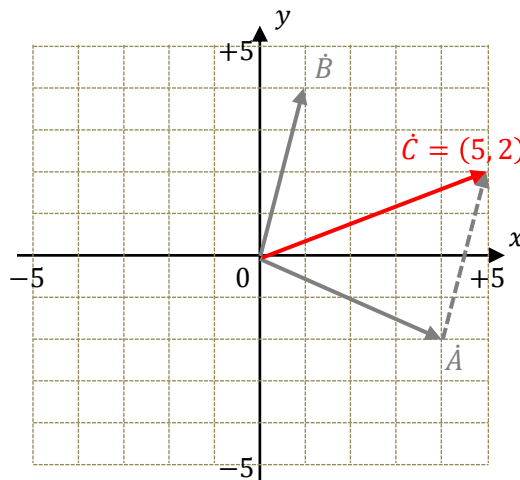


練習問題2

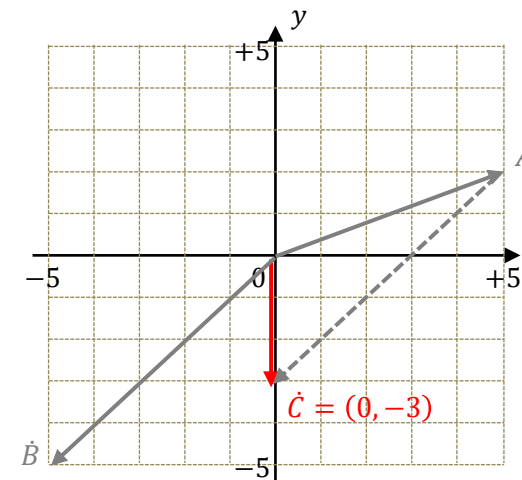
(1)
 $\dot{C} = \dot{A} + \dot{B}$



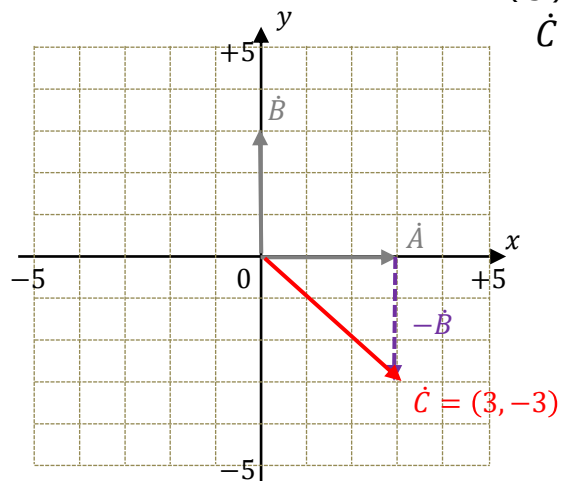
(2)
 $\dot{C} = \dot{A} + \dot{B}$



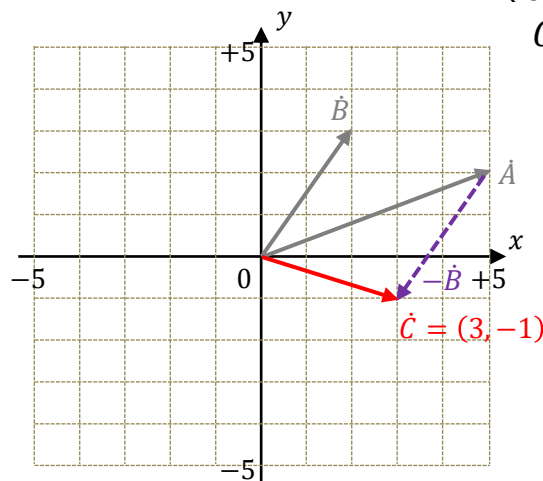
(3)
 $\dot{C} = \dot{A} + \dot{B}$



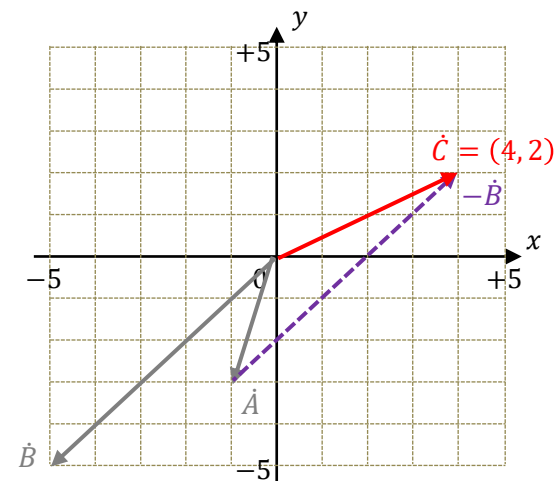
(4)
 $\dot{C} = \dot{A} - \dot{B}$



(5)
 $\dot{C} = \dot{A} - \dot{B}$



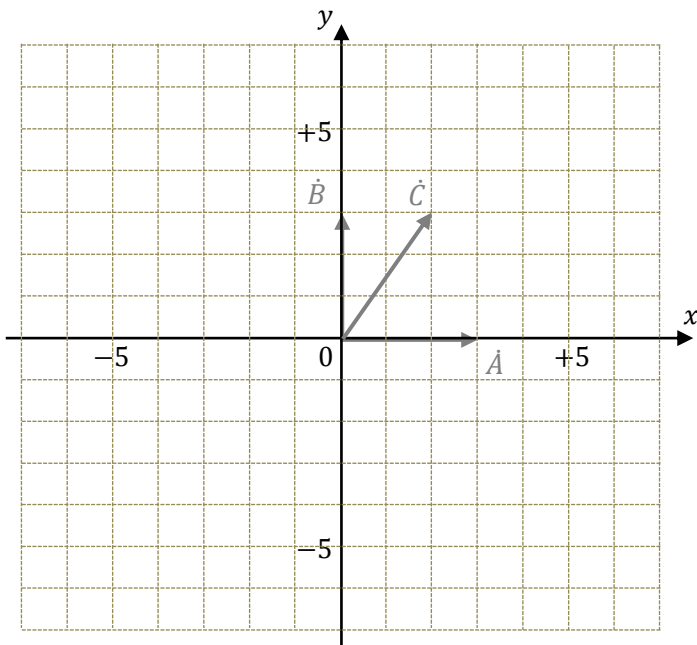
(6)
 $\dot{C} = \dot{A} - \dot{B}$



練習問題3

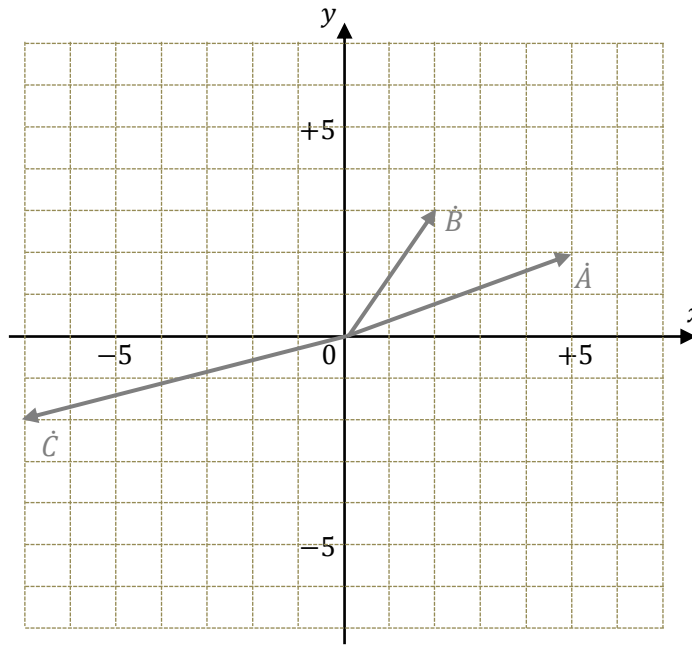
(1)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



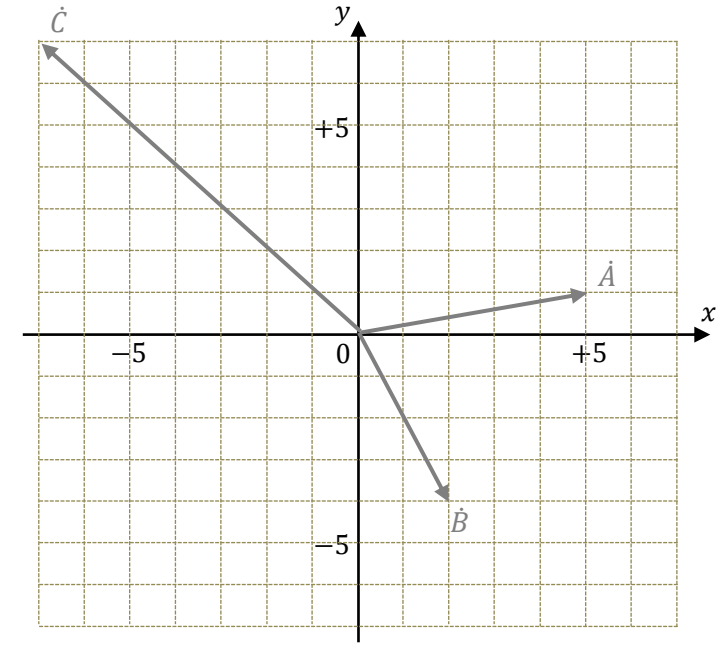
(2)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



(3)

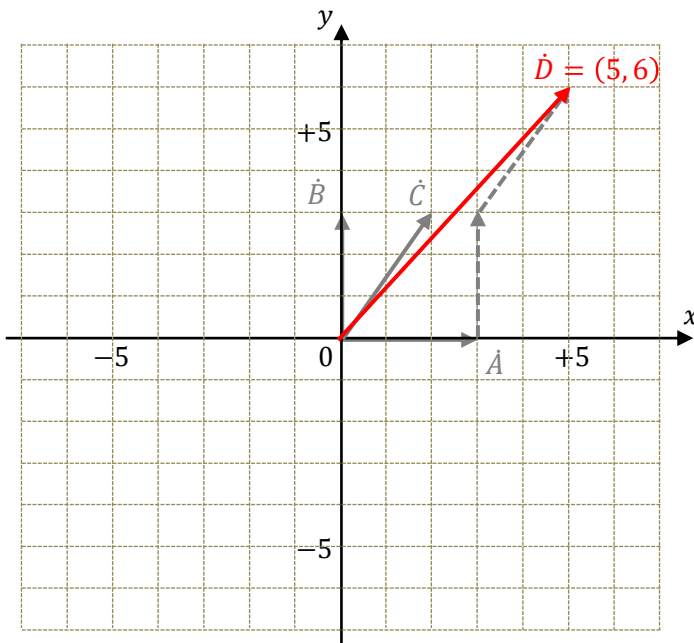
$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



練習問題3

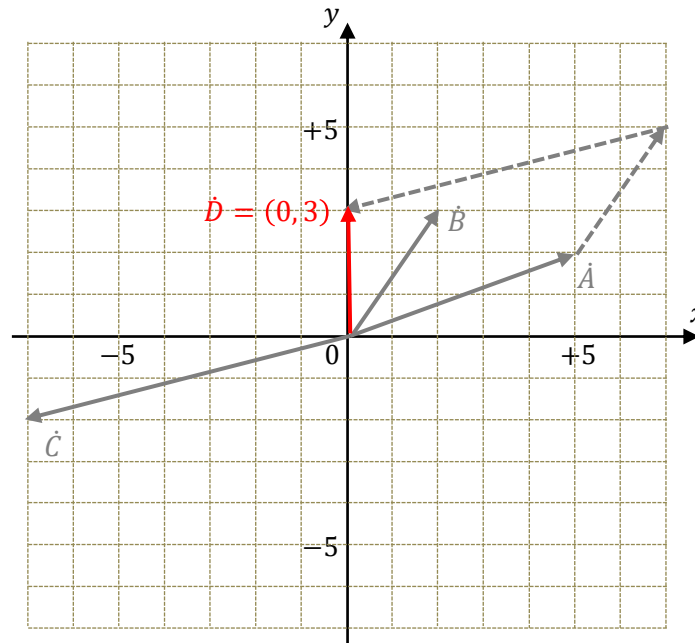
(1)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



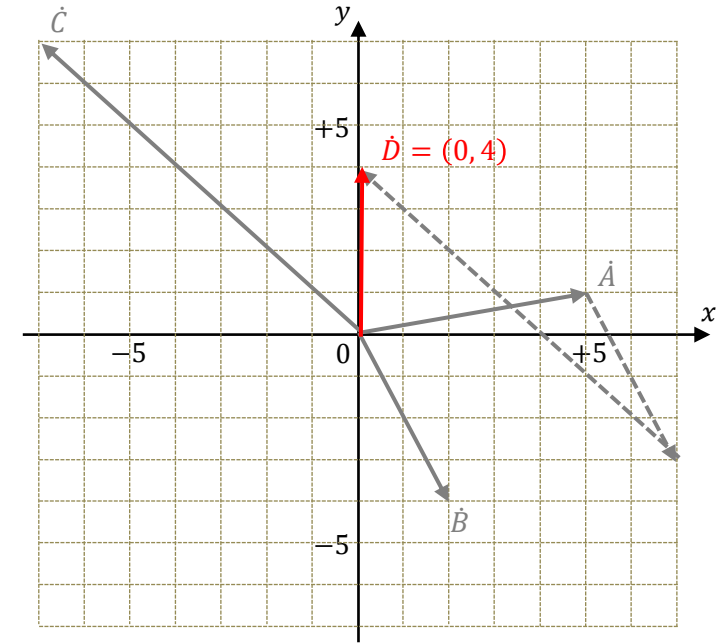
(2)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



(3)

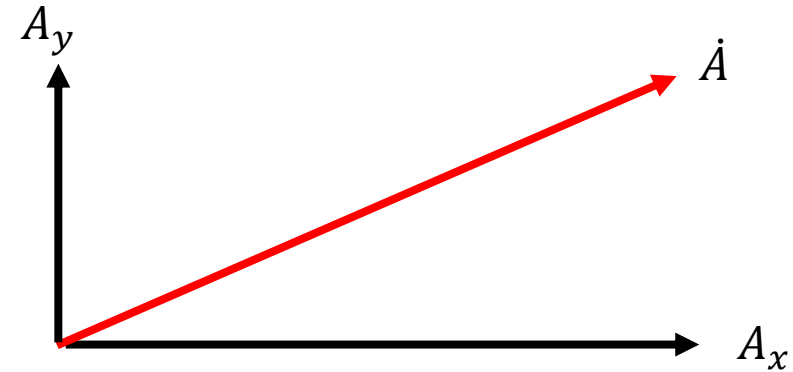
$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$



ベクトルの大きさと成分分解

\vec{A} の大きさ $A, |\vec{A}|$ などと表記

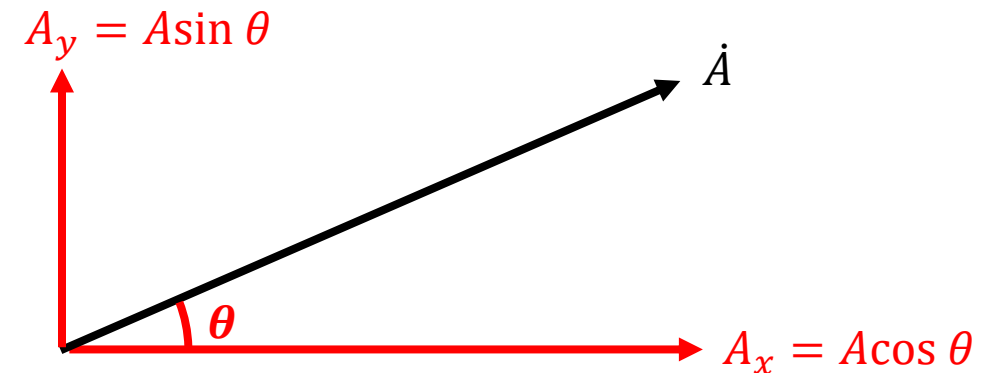
$$A = \sqrt{A_x^2 + A_y^2}$$



\vec{A} の成分分解

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$



練習問題4

(1) \vec{A} の大きさ

(2) \vec{B} の大きさ

Ans. $A =$ _____

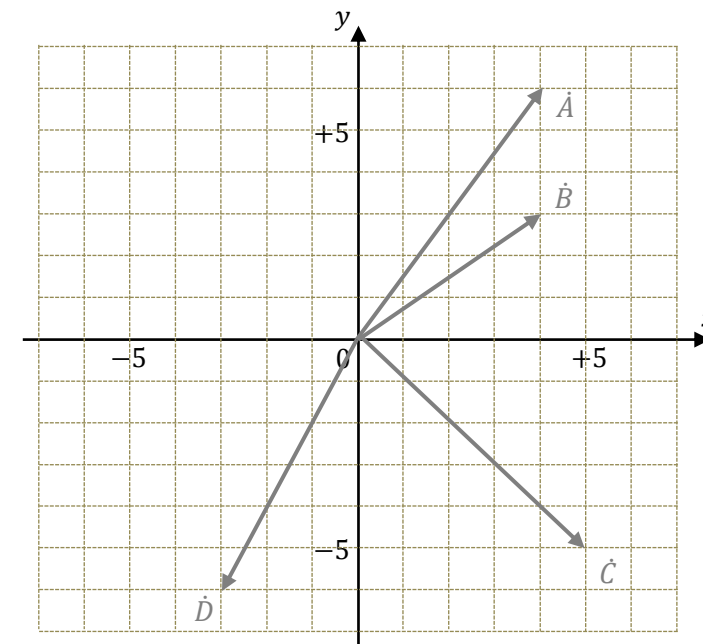
Ans. $B =$ _____

(3) \vec{C} の大きさ

(4) \vec{D} の大きさ

Ans. $C =$ _____

Ans. $D =$ _____



練習問題4

(1) \vec{A} の大きさ

$$\begin{aligned} A &= \sqrt{4^2 + 6^2} = \sqrt{16 + 36} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

Ans. $A = 2\sqrt{13}$

(2) \vec{B} の大きさ

$$\begin{aligned} B &= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

Ans. $B = 5$

(3) \vec{C} の大きさ

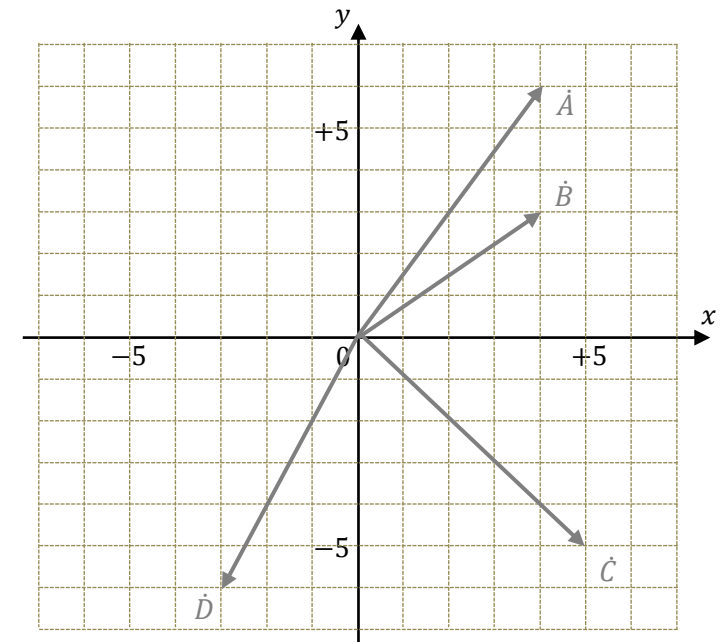
$$\begin{aligned} C &= \sqrt{5^2 + (-5)^2} = \sqrt{25 + 25} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

Ans. $C = 5\sqrt{2}$

(4) \vec{D} の大きさ

$$\begin{aligned} D &= \sqrt{(-3)^2 + (-6)^2} = \sqrt{9 + 36} \\ &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

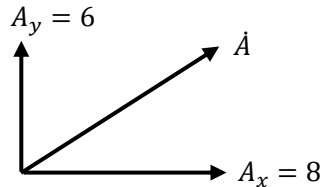
Ans. $D = 3\sqrt{5}$



練習問題5

(1)

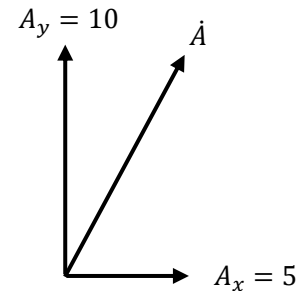
$A =$



Ans. $A =$ _____

(2)

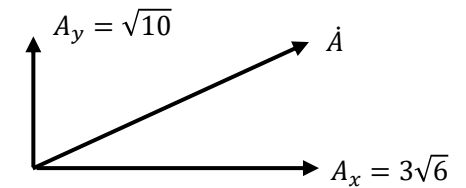
$A =$



Ans. $A =$ _____

(3)

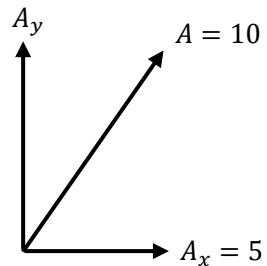
$A =$



Ans. $A =$ _____

(4)

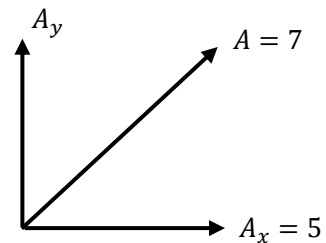
$A_y =$



Ans. $A_y =$ _____

(5)

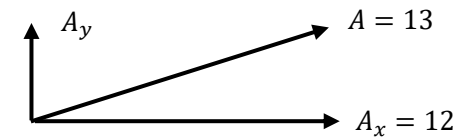
$A_y =$



Ans. $A_y =$ _____

(6)

$A_y =$

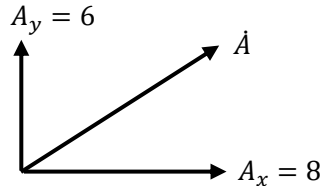


Ans. $A_y =$ _____

練習問題5

(1)

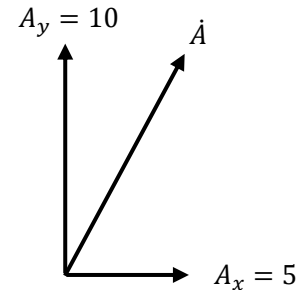
$$\begin{aligned} A &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} = 10 \end{aligned}$$



Ans. $A = 10$

(2)

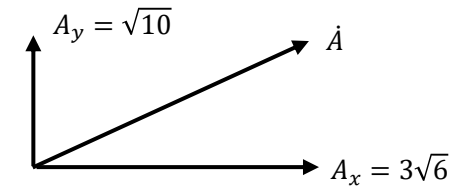
$$\begin{aligned} A &= \sqrt{5^2 + 10^2} \\ &= 5\sqrt{1^2 + 2^2} \\ &= 5\sqrt{5} \end{aligned}$$



Ans. $A = 5\sqrt{5}$

(3)

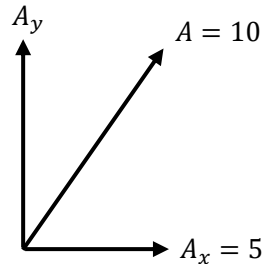
$$\begin{aligned} A &= \sqrt{(3\sqrt{6})^2 + (\sqrt{10})^2} \\ &= \sqrt{3^2 \times 6 + 10} \\ &= \sqrt{54 + 10} = \sqrt{64} = 8 \end{aligned}$$



Ans. $A = 8$

(4)

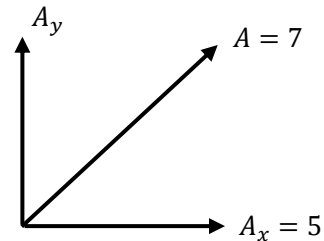
$$\begin{aligned} A_y &= \sqrt{10^2 - 5^2} \\ &= \sqrt{100 - 25} \\ &= \sqrt{75} = 5\sqrt{3} \end{aligned}$$



Ans. $A_y = 5\sqrt{3}$

(5)

$$\begin{aligned} A_y &= \sqrt{7^2 - 5^2} \\ &= \sqrt{49 - 25} \\ &= \sqrt{24} = 2\sqrt{6} \end{aligned}$$



Ans. $A_y = 2\sqrt{6}$

(6)

$$\begin{aligned} A_y &= \sqrt{13^2 - 12^2} \\ &= \sqrt{169 - 144} \\ &= \sqrt{25} = 5 \end{aligned}$$

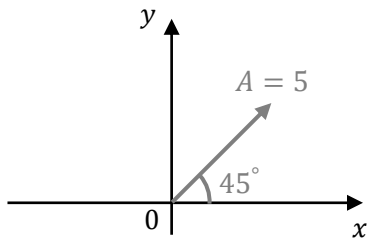


Ans. $A_y = 5$

練習問題6

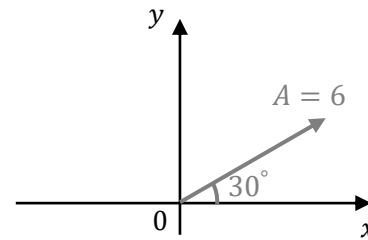
A_x, A_y を求めよ

(1)



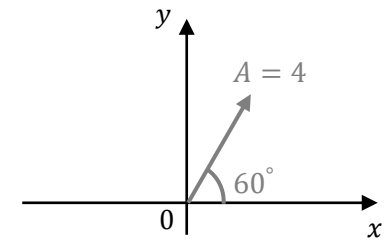
Ans. $A_x =$ $A_y =$

(2)



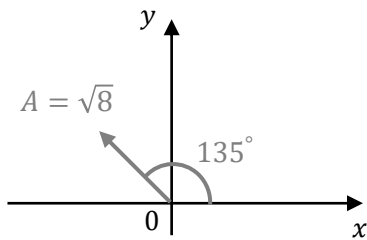
Ans. $A_x =$ $A_y =$

(3)



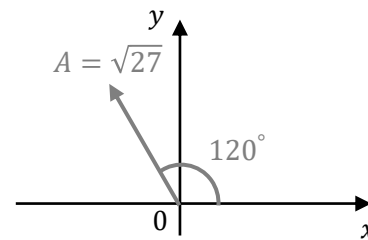
Ans. $A_x =$ $A_y =$

(4)



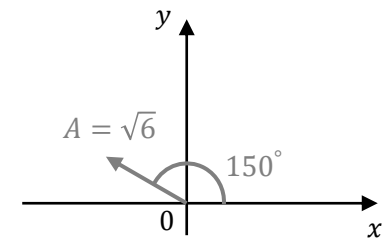
Ans. $A_x =$ $A_y =$

(5)



Ans. $A_x =$ $A_y =$

(6)



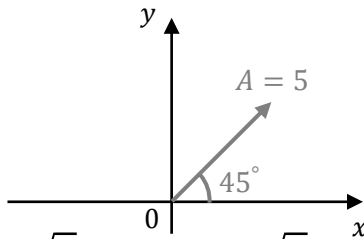
Ans. $A_x =$ $A_y =$

練習問題6

A_x, A_y を求めよ

(1)

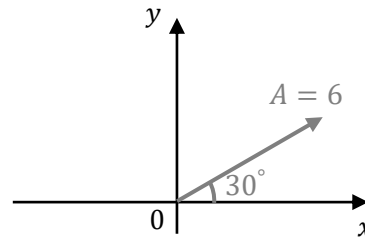
$$\begin{aligned} A_x &= A \cos 45^\circ \\ &= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \\ A_y &= A \sin 45^\circ \\ &= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \end{aligned}$$



Ans. $A_x = \frac{5\sqrt{2}}{2}$ $A_y = \frac{5\sqrt{2}}{2}$

(2)

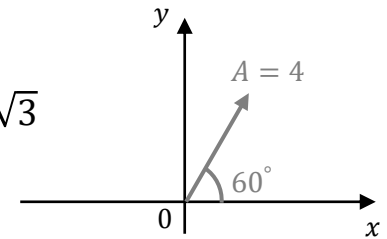
$$\begin{aligned} A_x &= A \cos 30^\circ \\ &= 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \\ A_y &= A \sin 30^\circ \\ &= 6 \times \frac{1}{2} = 3 \end{aligned}$$



Ans. $A_x = 3\sqrt{3}$ $A_y = 3$

(3)

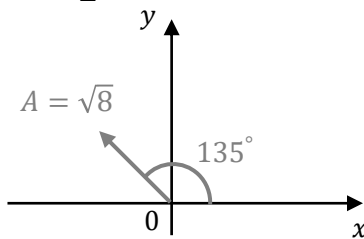
$$\begin{aligned} A_x &= A \cos 60^\circ \\ &= 4 \times \frac{1}{2} = 2 \\ A_y &= A \sin 60^\circ \\ &= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} \end{aligned}$$



Ans. $A_x = 2$ $A_y = 2\sqrt{3}$

(4)

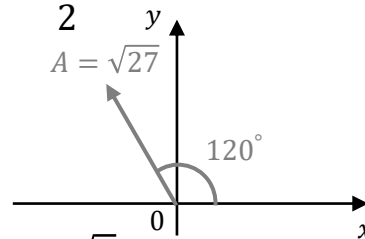
$$\begin{aligned} A_x &= A \cos 135^\circ \\ &= \sqrt{8} \times \left(-\frac{1}{\sqrt{2}}\right) = -2 \\ A_y &= A \sin 135^\circ \\ &= \sqrt{8} \times \frac{1}{\sqrt{2}} = 2 \end{aligned}$$



Ans. $A_x = -2$ $A_y = 2$

(5)

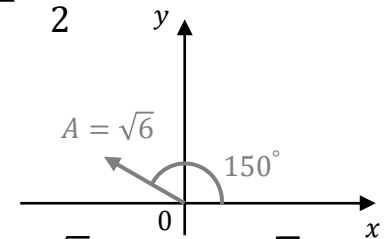
$$\begin{aligned} A_x &= A \cos 120^\circ \\ &= \sqrt{27} \times \left(-\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} \\ A_y &= A \sin 30^\circ \\ &= \sqrt{27} \times \frac{\sqrt{3}}{2} = \frac{9}{2} \end{aligned}$$



Ans. $A_x = -\frac{3\sqrt{3}}{2}$ $A_y = \frac{9}{2}$

(6)

$$\begin{aligned} A_x &= A \cos 150^\circ \\ &= \sqrt{6} \times \left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{2}}{2} \\ A_y &= A \sin 150^\circ \\ &= \sqrt{6} \times \frac{1}{2} = \frac{\sqrt{6}}{2} \end{aligned}$$



Ans. $A_x = \frac{3\sqrt{2}}{2}$ $A_y = \frac{\sqrt{6}}{2}$

ベクトルの合成

$$A_x = A \cos \theta_1$$

$$A_y = A \sin \theta_1$$

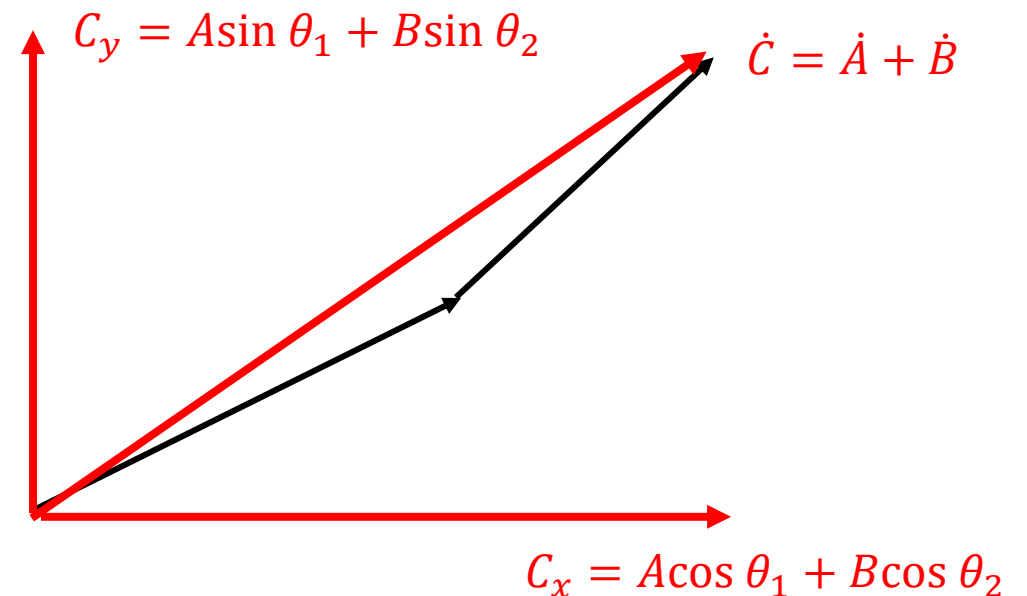
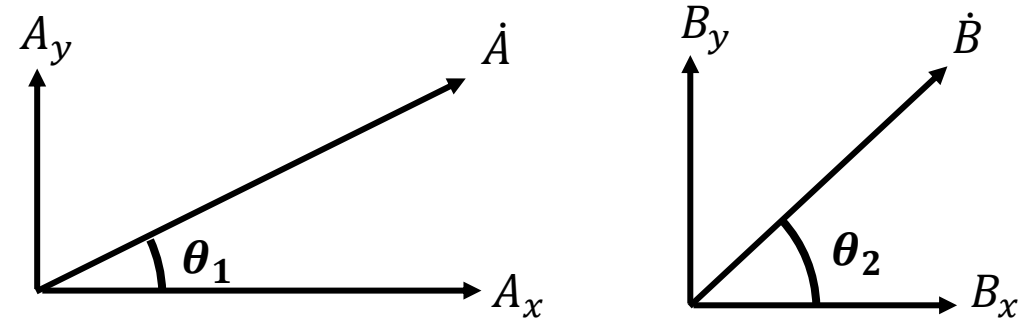
$$B_x = B \cos \theta_2$$

$$B_y = B \sin \theta_2$$

$$C_x = A \cos \theta_1 + B \cos \theta_2$$

$$C_y = A \sin \theta_1 + B \sin \theta_2$$

$$C = A + B = \sqrt{C_x^2 + C_y^2}$$



練習問題7

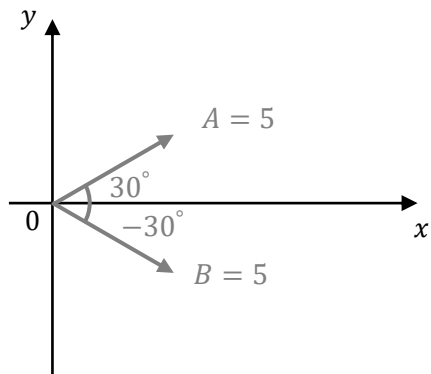
$A_x + B_x, A_y + B_y$ を求めよ



(1)

$$A_x + B_x =$$

$$A_y + B_y =$$

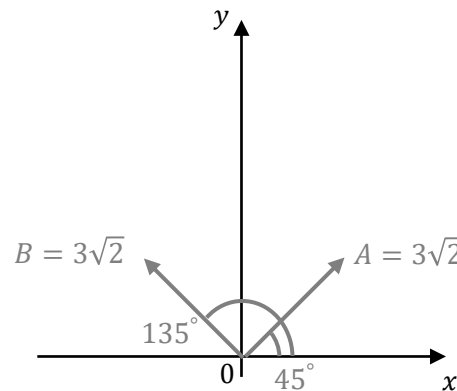


$$\text{Ans. } \begin{aligned} A_x + B_x &= \\ A_y + B_y &= \end{aligned}$$

(2)

$$A_x + B_x =$$

$$A_y + B_y =$$

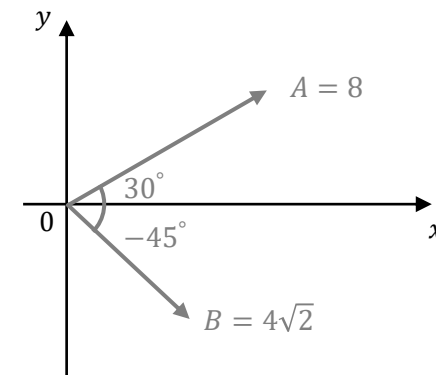


$$\text{Ans. } \begin{aligned} A_x + B_x &= \\ A_y + B_y &= \end{aligned}$$

(3)

$$A_x + B_x =$$

$$A_y + B_y =$$



$$\text{Ans. } \begin{aligned} A_x + B_x &= \\ A_y + B_y &= \end{aligned}$$

練習問題7

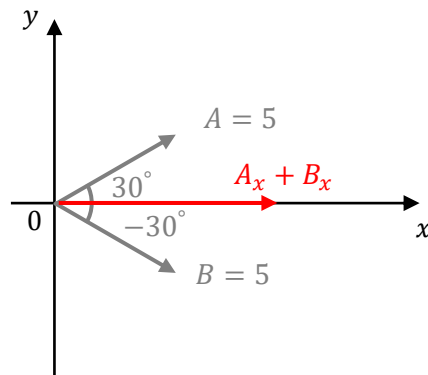
$A_x + B_x$, $A_y + B_y$ を求めよ



(1)

$$\begin{aligned} A_x + B_x &= A \cos 30^\circ + B \cos(-30^\circ) \\ &= 5 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \end{aligned}$$

$$\begin{aligned} A_y + B_y &= A \sin 30^\circ + B \sin(-30^\circ) \\ &= 5 \times \frac{1}{2} + 5 \times \left(-\frac{1}{2}\right) = 0 \end{aligned}$$

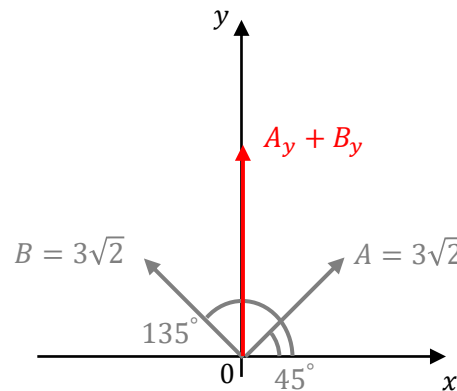


Ans. $A_x + B_x = 10\sqrt{3}$
 $A_y + B_y = 0$

(2)

$$\begin{aligned} A_x + B_x &= A \cos 45^\circ + B \cos 135^\circ \\ &= 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 3\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} A_y + B_y &= A \sin 30^\circ + B \sin(-30^\circ) \\ &= 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 3\sqrt{2} \times \frac{1}{\sqrt{2}} = 6 \end{aligned}$$

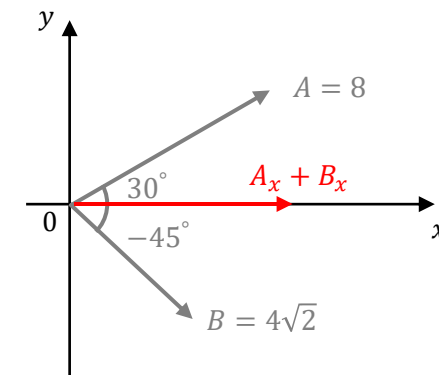


Ans. $A_x + B_x = 0$
 $A_y + B_y = 6$

(3)

$$\begin{aligned} A_x + B_x &= A \cos 30^\circ + B \cos(-45^\circ) \\ &= 8 \times \frac{\sqrt{3}}{2} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} = 10\sqrt{3} \\ &= 4\sqrt{3} + 4 \end{aligned}$$

$$\begin{aligned} A_y + B_y &= A \sin 30^\circ + B \sin(-45^\circ) \\ &= 8 \times \frac{1}{2} + 4\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) = 0 \end{aligned}$$



Ans. $A_x + B_x = 4\sqrt{3} + 4$
 $A_y + B_y = 0$

練習問題8

$A + B$ を求めよ

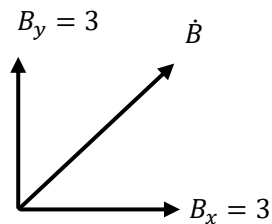
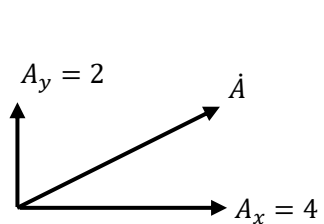


(1)

$$A_x + B_x =$$

$$A_y + B_y =$$

$$A + B =$$



Ans. $A + B =$ _____

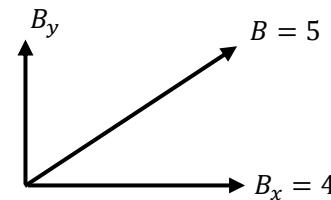
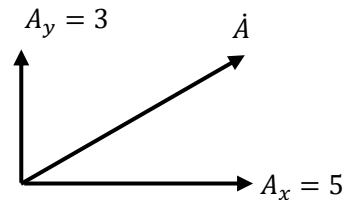
(2)

$$A_x + B_x =$$

$$B_y =$$

$$A_y + B_y =$$

$$A + B =$$



Ans. $A + B =$ _____

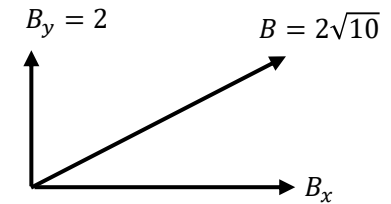
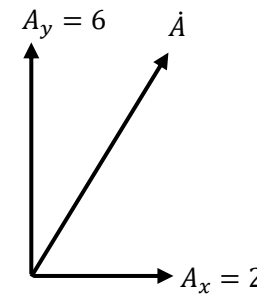
(3)

$$B_x =$$

$$A_x + B_x =$$

$$A_y + B_y =$$

$$A + B =$$



Ans. $A + B =$ _____

練習問題8

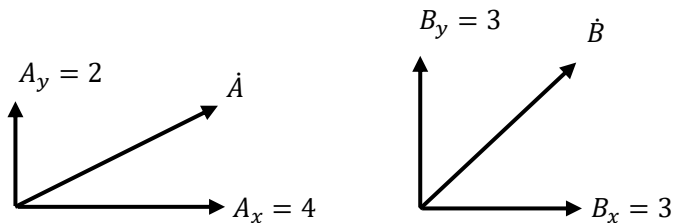
A + Bを求めよ

(1)

$$A_x + B_x = 4 + 3 = 7$$

$$A_y + B_y = 2 + 3 = 5$$

$$\begin{aligned} A + B &= \sqrt{7^2 + 5^2} \\ &= \sqrt{49 + 25} = \sqrt{74} \end{aligned}$$



Ans. $A + B = \sqrt{74}$

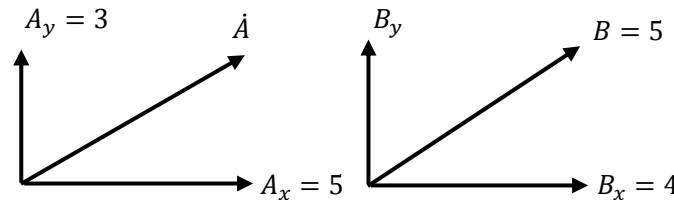
(2)

$$A_x + B_x = 5 + 4 = 9$$

$$\begin{aligned} B_y &= \sqrt{5^2 - 4^2} = \sqrt{25 - 16} \\ &= \sqrt{9} = 3 \end{aligned}$$

$$A_y + B_y = 3 + 3 = 6$$

$$\begin{aligned} A + B &= \sqrt{9^2 + 6^2} \\ &= \sqrt{81 + 36} = \sqrt{117} \\ &= \sqrt{9 \times 13} = 3\sqrt{13} \end{aligned}$$



Ans. $A + B = 3\sqrt{13}$

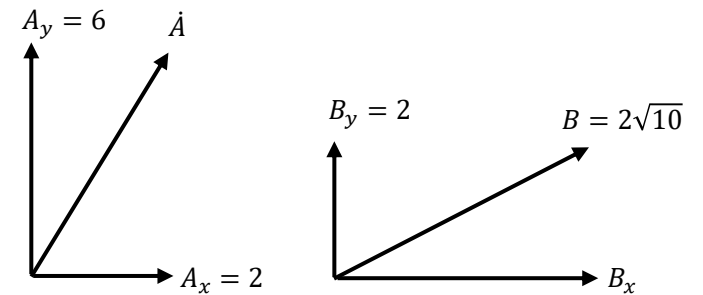
(3)

$$\begin{aligned} B_x &= \sqrt{(2\sqrt{10})^2 - 4^2} = \sqrt{40 - 4} \\ &= \sqrt{36} = 6 \end{aligned}$$

$$A_x + B_x = 2 + 6 = 8$$

$$A_y + B_y = 6 + 2 = 8$$

$$\begin{aligned} A + B &= \sqrt{8^2 + 8^2} \\ &= \sqrt{8^2 \times 2} = 8\sqrt{2} \end{aligned}$$



Ans. $A + B = 8\sqrt{2}$

ご聴講ありがとうございました!!