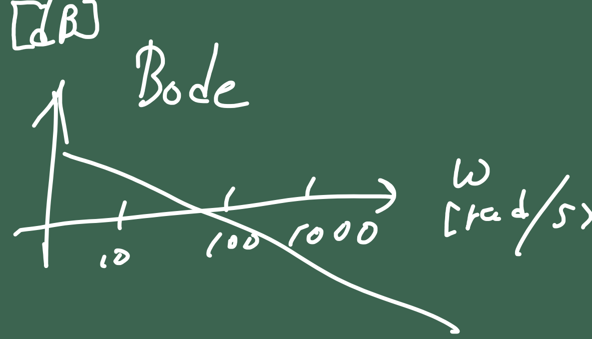
二種への複素数

制御：log

H30 問4.



準備

1 $\mathcal{L}[f(t)] = F(s)$

a) $\mathcal{L}\left[\frac{df(t)}{dt}\right] = ?$

b) $\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = ?$

2 $G_1(s) = \frac{10}{s}$, $G_2(s) = \frac{1}{s^2}$

$G(s) = G_1(s)\,G_2(s)$ のボード線図のゲイン特性を描け

1 $\mathcal{L}[f(t)] = F(s)$

a) $\mathcal{L}\left[\frac{df(t)}{dt}\right] = ?$

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答 $\mathcal{L}[f(t)] = \int_0^\infty f(t)\,e^{-st}\,dt$

$$\mathcal{L}\left[\frac{df}{dt}(t)\right] = \int_0^\infty \frac{df(t)}{dt}\,e^{-st}\,dt$$ 部分積分

$$= \left[f(t)\,e^{-st}\right]_0^\infty - \int_0^\infty f(t)\left(e^{-st}\right)'\,dt$$

$$= f(\infty)\,e^{-s\infty} - f(0)\,e^{-s0} - \int_0^\infty f(t)\,(-s)\,e^{-st}\,dt$$

($f(\infty)e^{-s\infty} \to 0$, $e^{-s0} = 1$, $f(0)$)

$$= -f(0) + s\int_0^\infty f(t)\,e^{-st}\,dt$$

$$= -f(0) + s\,\mathcal{L}[f(t)]$$

($\mathcal{L}[f(t)] = F(s)$)

$$= -f(0) + sF(s)$$

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right]$$

$$\int_0^t f(\tau)\,d\tau = g(t)$$

（微分積分学の基本定理）

$$\frac{dg(t)}{dt} = f(t)$$

$$\mathcal{L}\left[\frac{dg(t)}{dt}\right] = s\mathcal{L}[g(t)] - g(0) \qquad g(0) = 0$$

$$g(0) = \int_0^0 f(\tau)\,d\tau = 0$$

$$\mathcal{L}\left[\frac{dg(t)}{dt}\right] = s\mathcal{L}[g(t)]$$

$$\mathcal{L}[g(t)] = \frac{1}{s}\mathcal{L}\left[\frac{dg(t)}{dt}\right] = \frac{1}{s}\mathcal{L}[f(t)]$$

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{1}{s}\mathcal{L}[f(t)] = \frac{1}{s}F(s)$$

2 $G_1(s) = \frac{10}{s}$, $G_2(s) = \frac{1}{s^2}$

$G(s) = G_1(s)\, G_2(s)$ のボード線図のゲイン特性を描け

$$G(s) = \frac{10}{s^3} \qquad G(j\omega) = \frac{10}{j^3 \omega^3}$$

ゲイン $20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{10}{j^3\omega^3}\right|$

$$= 20\log_{10} 10 - 20\log_{10}\omega^3$$

$\underbrace{\log_{10} 10}_{1}$ $\qquad |j^3| = 1$

$$= 20 \cdot 1 - 20 \cdot 3\log_{10}\omega$$

$$= 20 - 60\log_{10}\omega$$

G_1 $20\log_{10}|G_1(j\omega)| = 20\log_{10}\left|\frac{10}{j\omega}\right|$

$$= 20\log_{10}10 - 20\log_{10}\omega$$

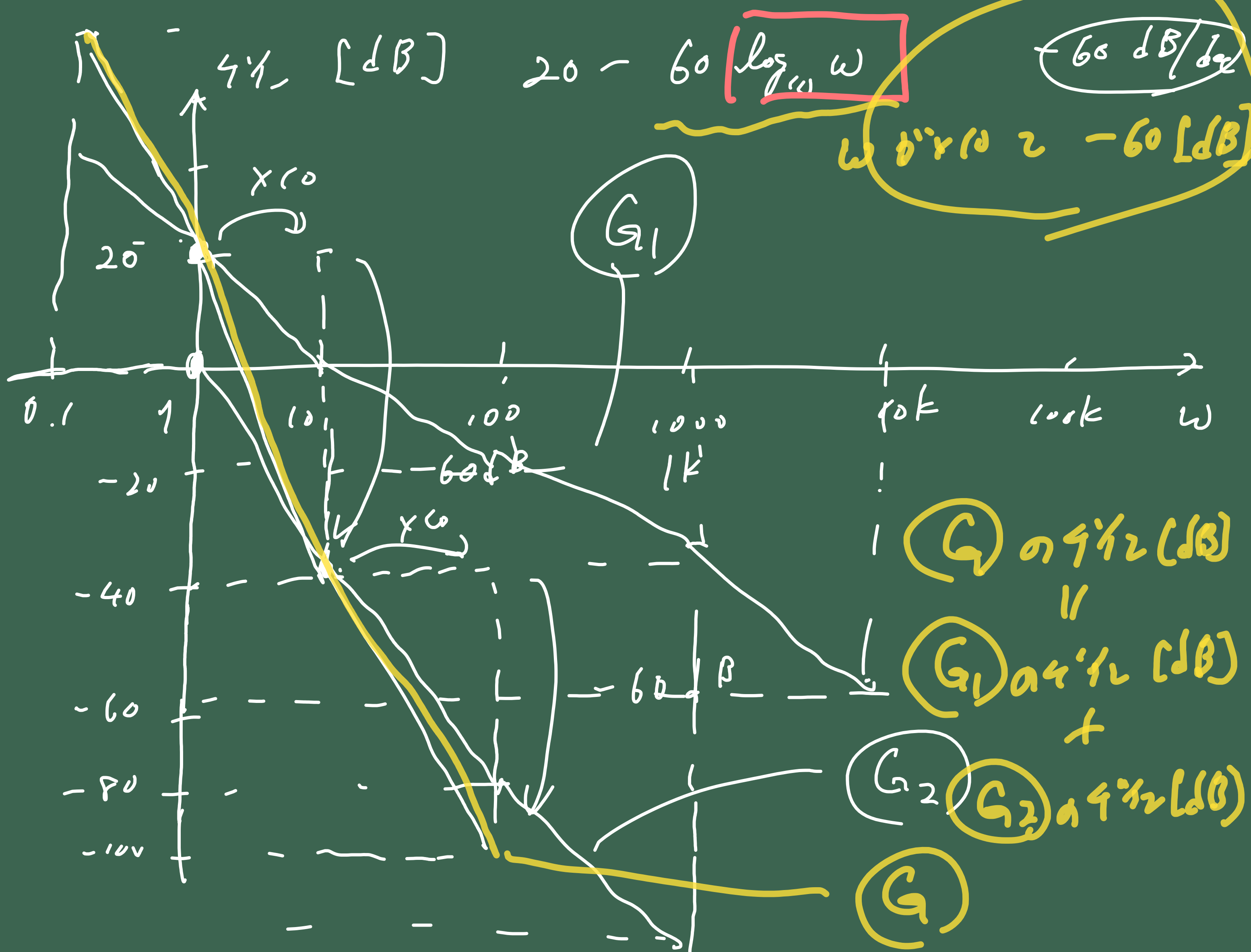
$$= 20 - 20\log_{10}\omega$$

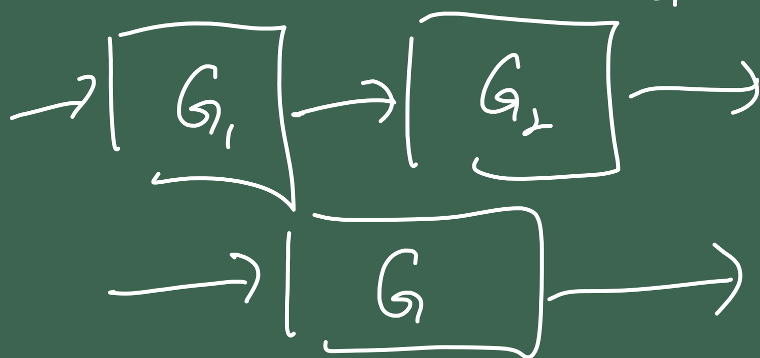
-20 dB/dec

G_2 $20\log_{10}|G_2(j\omega)| = 20\log_{10}\left|\frac{1}{j^2\omega^2}\right|$

$$= 20\log_{10}1 - 20\log_{10}\omega^2 = 0 - 20 \cdot 2\log_{10}\omega = -40\log_{10}\omega$$

+ -40 dB/dec





$$G(s) = G_1(s)\,G_2(s)$$

$$20\log_{10}|G(j\omega)| = 20\log_{10}|G_1(j\omega)\,G_2(j\omega)|$$

$$= 20\log_{10}\left(|G_1(j\omega)|\,|G_2(j\omega)|\right)$$

$$|\dot{Z}_1 \cdot \dot{Z}_2| = |\dot{Z}_1|\,|\dot{Z}_2|$$

$$= \underbrace{20\log_{10}|G_1(j\omega)|}_{G_1のゲイン} + \underbrace{20\log_{10}|G_2(j\omega)|}_{G_2のゲイン}$$

和