

電験三種 オンライン講座

電気数学 第11回 三角関数

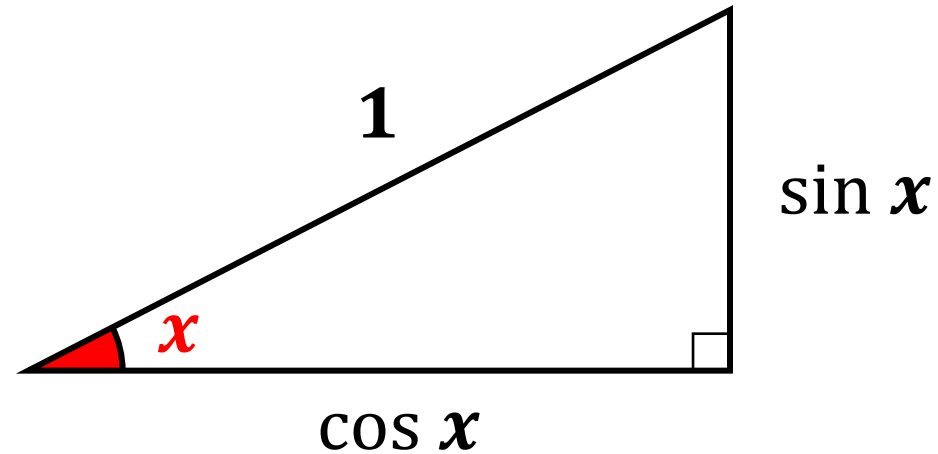
三角関数

直角三角形の1つの角を変数 x で表し、 x に対する三角比の値を y とした関数を“三角関数”という

$$y = \sin x$$

$$y = \cos x$$

$$y = \tan x$$



<三角関数に慣れるために必要な数学の土台>

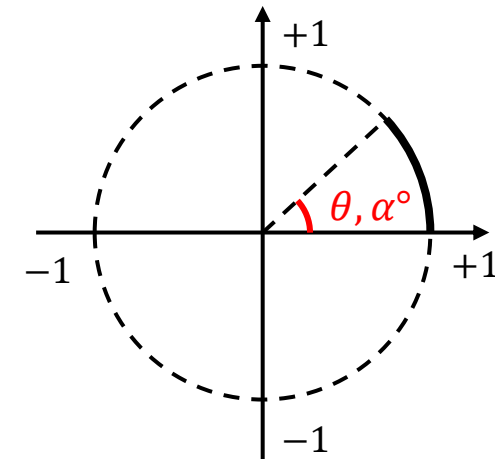
- 度数法と弧度法による角度の表現
- 三平方の定理
- 三角比

度数法と弧度法

○度数法と弧度法

角の大きさを表す際に、度 [°] を用いる方法を度数法という。

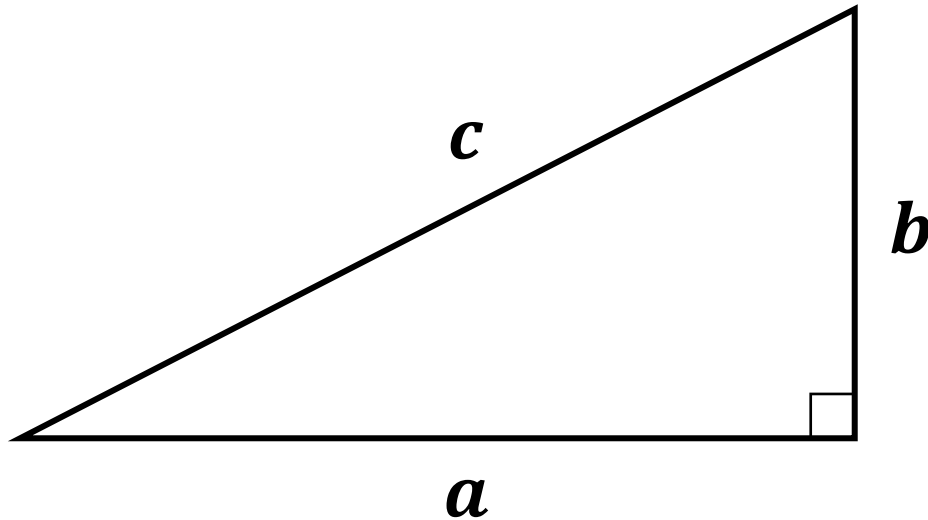
半径1の単位円の円周の長さで角を表す方法を弧度法といい、この角の大きさはラジアン [rad] という単位を用いる。円の1周の角360°を弧度法で表すと2π [rad] となる。



$$\theta \text{ [rad]} = \alpha^\circ \times \frac{2\pi}{360^\circ}$$

度 [°]	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
ラジアン [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

三平方の定理



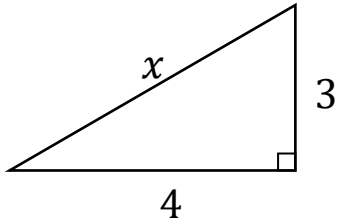
2辺の長さを a , b , 斜辺の長さを c とする
直角三角形において次式が成り立つ。

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$

- 直角三角形に対する公式
- 直角三角形の各辺の長さの関係を表す
- 直角を作る2つの辺の長さとして直角と向かい合う辺（斜辺）の関係を表す

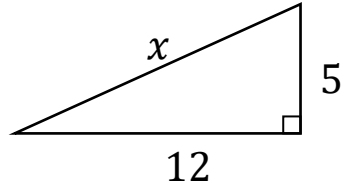
練習問題 I

(1)



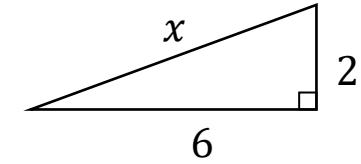
Ans. $x =$ _____

(2)



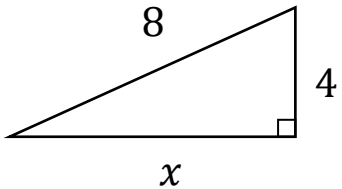
Ans. $x =$ _____

(3)



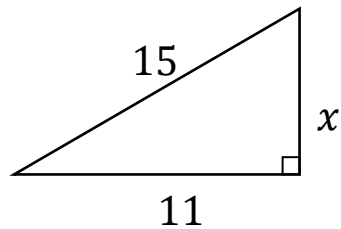
Ans. $x =$ _____

(4)



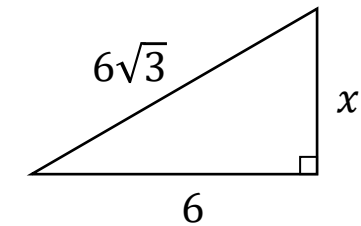
Ans. $x =$ _____

(5)



Ans. $x =$ _____

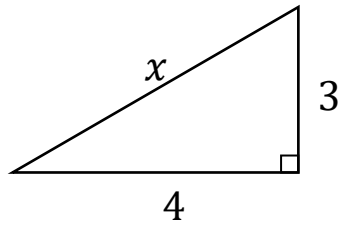
(6)



Ans. $x =$ _____

練習問題 I (解答)

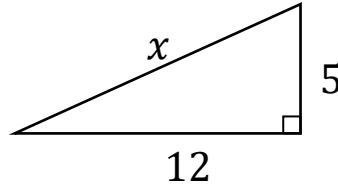
(1)



$$\begin{aligned}x^2 &= 3^2 + 4^2 = 25 \\x^2 &= 5^2 \\x &= 5\end{aligned}$$

Ans. $x = 5$

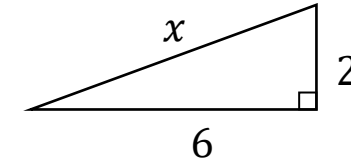
(2)



$$\begin{aligned}x^2 &= 5^2 + 12^2 = 169 \\x^2 &= 13^2 \\x &= 13\end{aligned}$$

Ans. $x = 13$

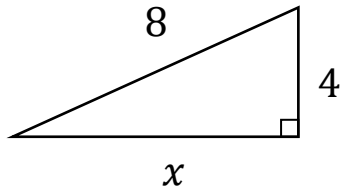
(3)



$$\begin{aligned}x^2 &= 2^2 + 6^2 \\&= 4 + 36 = 40 \\x &= \sqrt{40} = 2\sqrt{10}\end{aligned}$$

Ans. $x = 2\sqrt{10}$

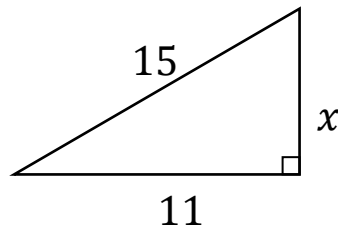
(4)



$$\begin{aligned}x^2 &= 8^2 - 4^2 \\&= 64 - 16 = 48 \\x &= \sqrt{48} = 4\sqrt{3}\end{aligned}$$

Ans. $x = 4\sqrt{3}$

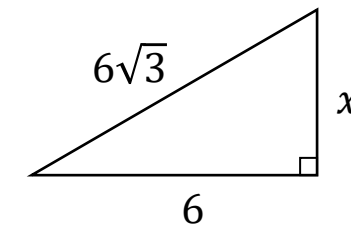
(5)



$$\begin{aligned}x^2 &= 15^2 - 11^2 \\x^2 &= 225 - 121 = 104 \\x &= \sqrt{104} = 2\sqrt{26}\end{aligned}$$

Ans. $x = 2\sqrt{26}$

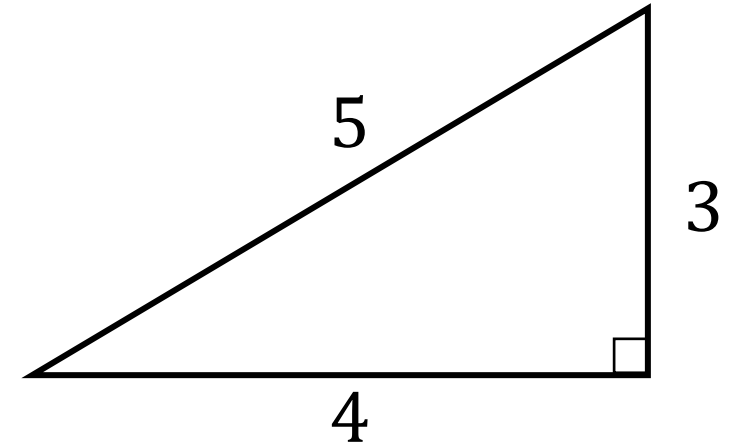
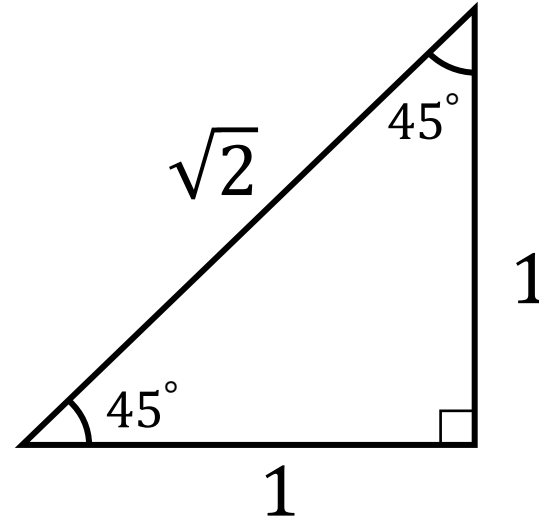
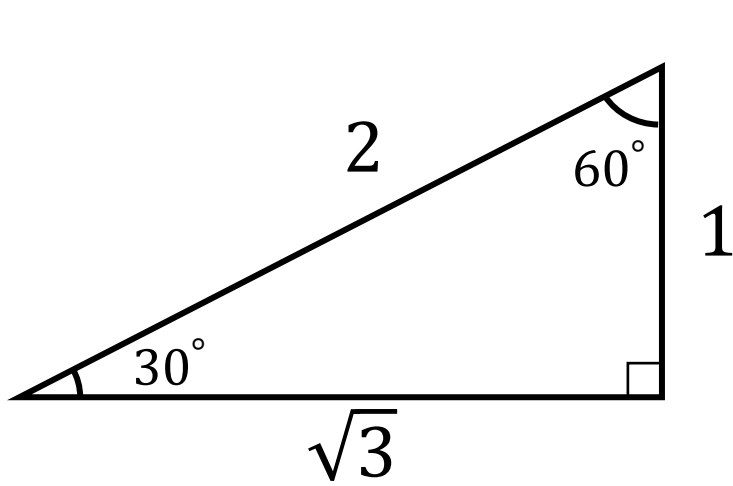
(6)



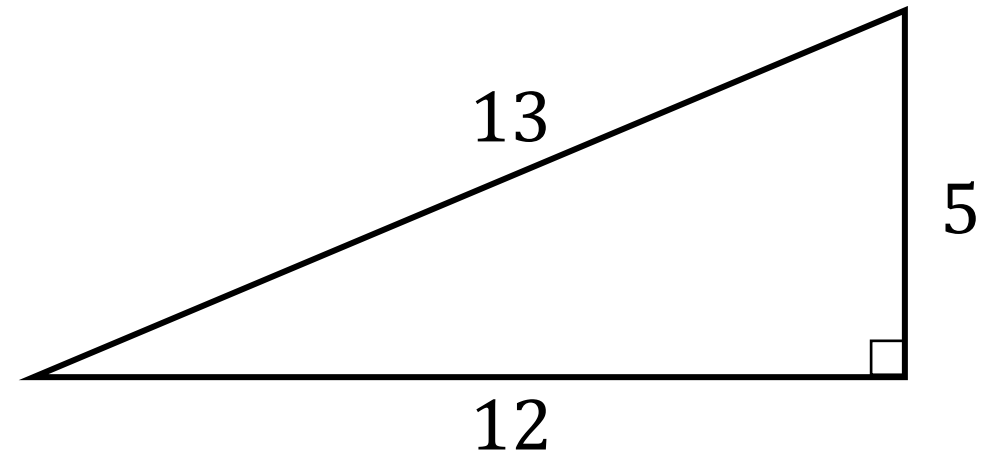
$$\begin{aligned}x^2 &= (6\sqrt{3})^2 - 6^2 \\x^2 &= 108 - 36 = 72 \\x &= \sqrt{72} = 6\sqrt{2}\end{aligned}$$

Ans. $x = 6\sqrt{2}$

特徴的な直角三角形

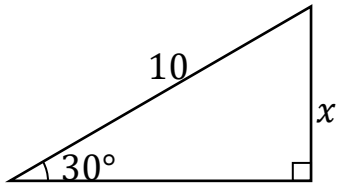


この4つの三角形の角度と辺の長さの比は全て覚えること!



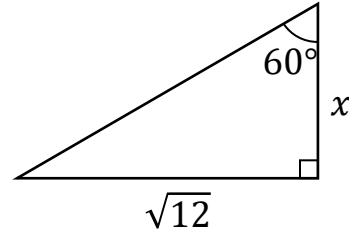
練習問題2

(1)



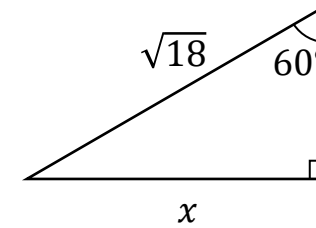
Ans. $x =$ _____

(2)



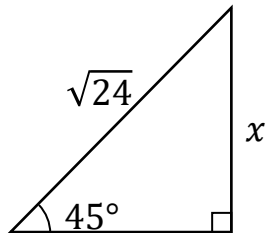
Ans. $x =$ _____

(3)



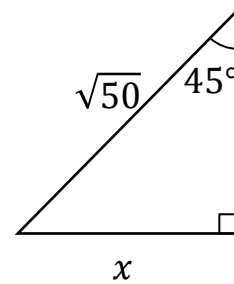
Ans. $x =$ _____

(4)



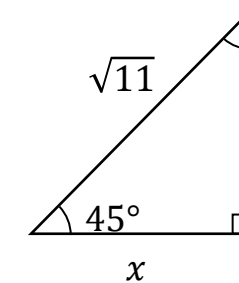
Ans. $x =$ _____

(5)



Ans. $x =$ _____

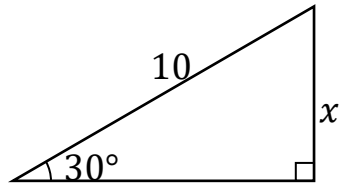
(6)



Ans. $x =$ _____

練習問題2 (解答)

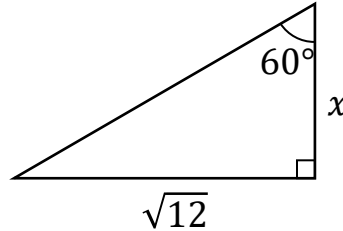
(1)



$$\begin{aligned} 1 : 2 &= x : 10 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Ans. $x = 5$

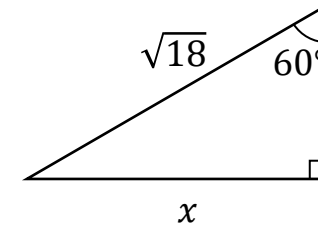
(2)



$$\begin{aligned} 1 : \sqrt{3} &= x : \sqrt{12} \\ \sqrt{3}x &= \sqrt{12} = 2\sqrt{3} \\ x &= 2 \end{aligned}$$

Ans. $x = 2$

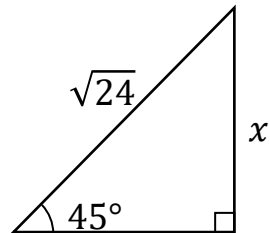
(3)



$$\begin{aligned} \sqrt{3} : 2 &= x : \sqrt{18} \\ 2x &= \sqrt{54} = 3\sqrt{6} \\ x &= \frac{3}{2}\sqrt{6} \end{aligned}$$

Ans. $x = \frac{3}{2}\sqrt{6}$

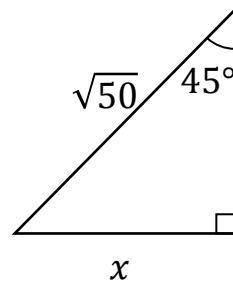
(4)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{24} \\ \sqrt{2}x &= \sqrt{24} \\ x &= \sqrt{12} = 2\sqrt{3} \end{aligned}$$

Ans. $x = 2\sqrt{3}$

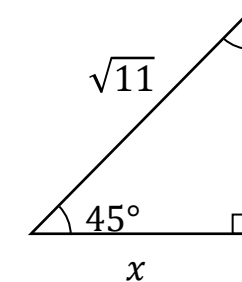
(5)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{50} \\ \sqrt{2}x &= \sqrt{50} = 5\sqrt{2} \\ x &= 5 \end{aligned}$$

Ans. $x = 5$

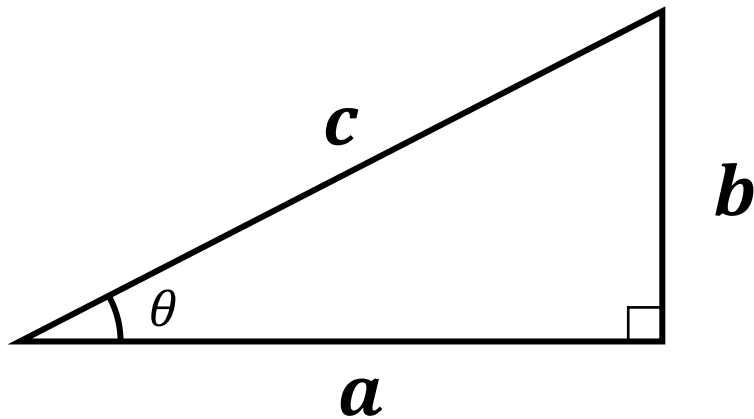
(6)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{11} \\ \sqrt{2}x &= \sqrt{11} \\ x &= \frac{\sqrt{11}}{\sqrt{2}} = \frac{\sqrt{22}}{2} \end{aligned}$$

Ans. $x = \frac{\sqrt{22}}{2}$

直角三角形と三角比



$$\sin \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}$$

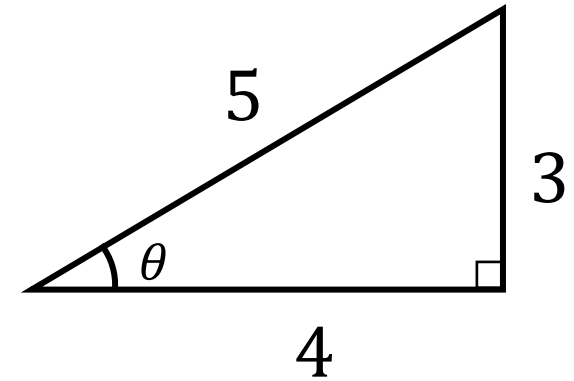
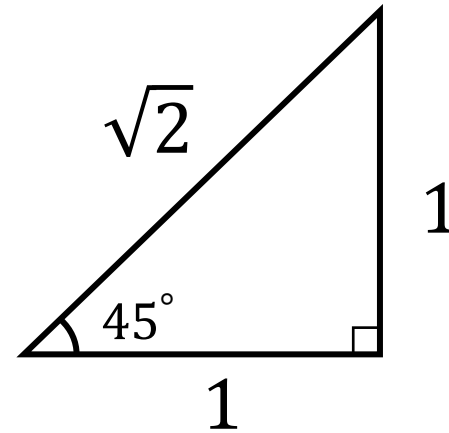
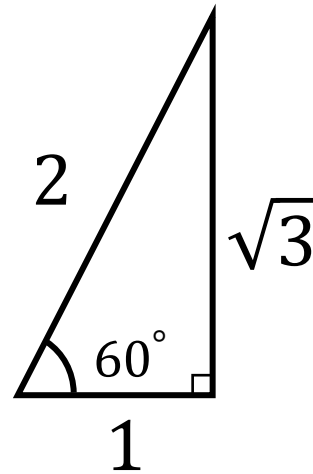
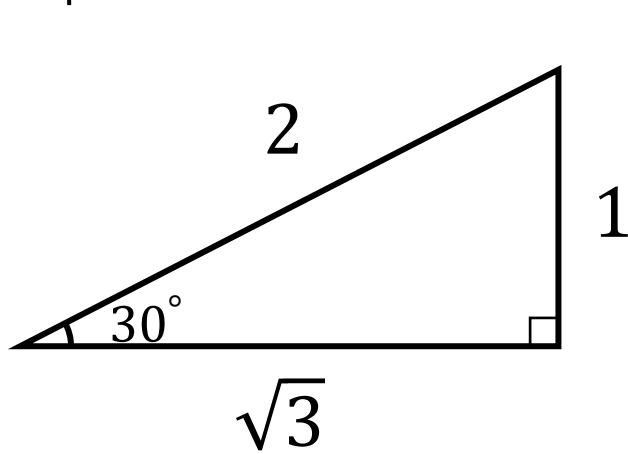
$$\cos \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\tan \theta = \frac{b}{a}$$

三角比は、直角三角形の
2辺の長さの比を表したもの

その比は角度 θ によって変化するが、
角度 θ が分からなければ値が導出できないわけではない

三角形と三角比



$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

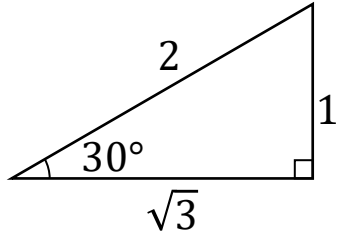
$$\tan 45^\circ = 1$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

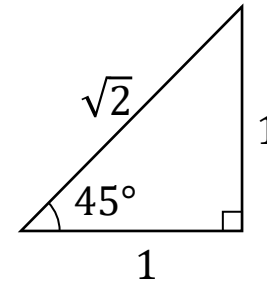
$$\tan \theta = \frac{3}{4}$$

練習問題3



(1) $\sin 60^\circ$

Ans. _____



(4) $\sin 45^\circ$

Ans. _____

(2) $\cos 60^\circ$

Ans. _____

(5) $\cos 45^\circ$

Ans. _____

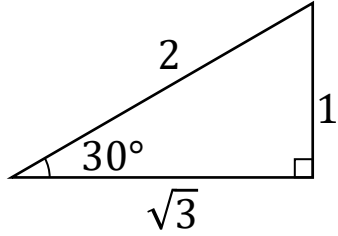
(3) $\tan 60^\circ$

Ans. _____

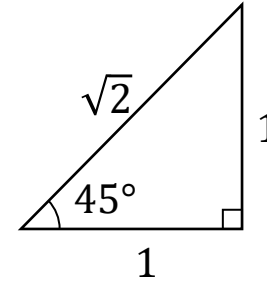
(6) $\tan 45^\circ$

Ans. _____

練習問題3 (解答)



(1) $\sin 60^\circ$ Ans. $\frac{\sqrt{3}}{2}$



(4) $\sin 45^\circ$ Ans. $\frac{1}{\sqrt{2}}$

(2) $\cos 60^\circ$ Ans. $\frac{1}{2}$

(5) $\cos 45^\circ$ Ans. $\frac{1}{\sqrt{2}}$

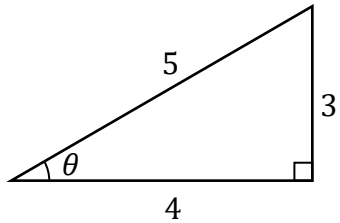
(3) $\tan 60^\circ$ Ans. $\sqrt{3}$

(6) $\tan 45^\circ$ Ans. 1

練習問題4

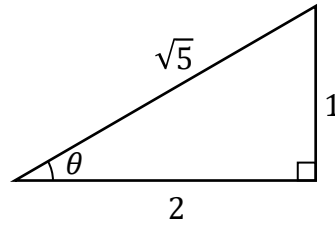


(1) $\sin \theta$



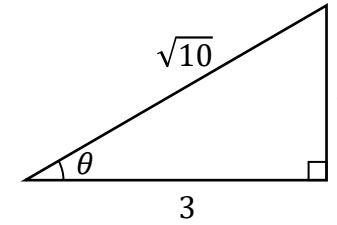
Ans. _____

(2) $\cos \theta$



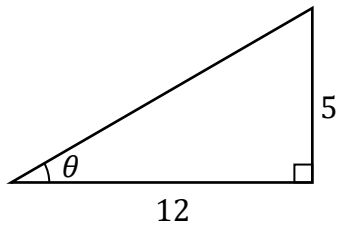
Ans. _____

(3) $\tan \theta$



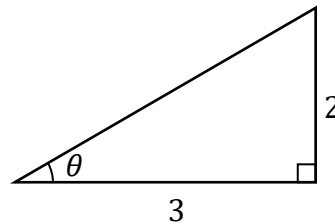
Ans. _____

(4) $\sin \theta$



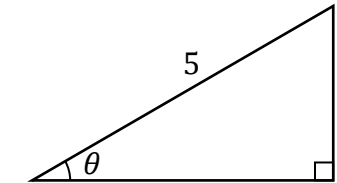
Ans. _____

(5) $\cos \theta$



Ans. _____

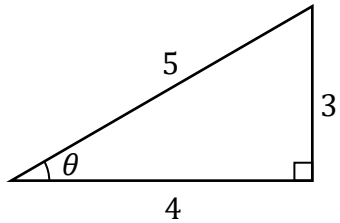
(6) $\tan \theta$



Ans. _____

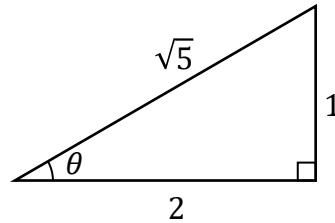
練習問題4 (解答)

(1) $\sin \theta$



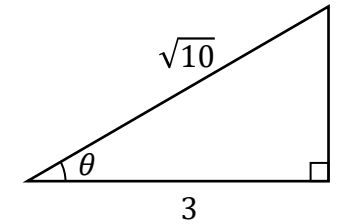
Ans. $\frac{3}{5}$

(2) $\cos \theta$



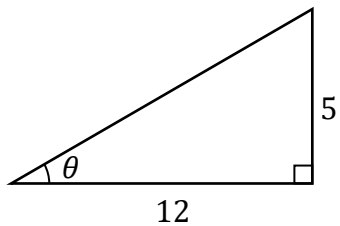
Ans. $\frac{2}{\sqrt{5}}$

(3) $\tan \theta$



Ans. $\frac{1}{3}$

(4) $\sin \theta$

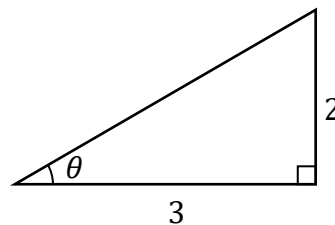


$$\sin \theta = \frac{5}{\sqrt{12^2 + 5^2}}$$

$$= \frac{5}{\sqrt{169}} = \frac{5}{13}$$

Ans. $\frac{5}{13}$

(5) $\cos \theta$



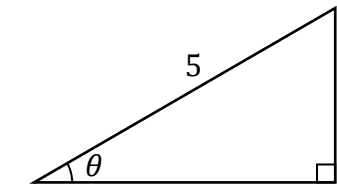
$$\cos \theta = \frac{3}{\sqrt{3^2 + 2^2}}$$

$$= \frac{3}{\sqrt{13}}$$

$$= \frac{3\sqrt{13}}{13}$$

Ans. $\frac{3\sqrt{13}}{13}$

(6) $\tan \theta$



$$\tan \theta = \frac{2}{\sqrt{5^2 - 2^2}}$$

$$= \frac{2}{\sqrt{21}}$$

$$= \frac{2\sqrt{21}}{21}$$

Ans. $\frac{2\sqrt{21}}{21}$

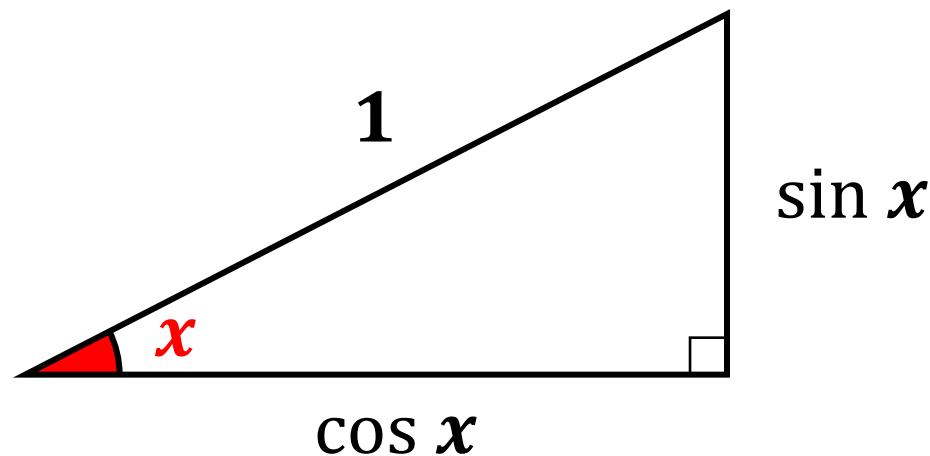
三角関数

直角三角形の1つの角を変数 x で表し、 x に対する三角比の値を y とした関数を“三角関数”という

$$y = \sin x$$

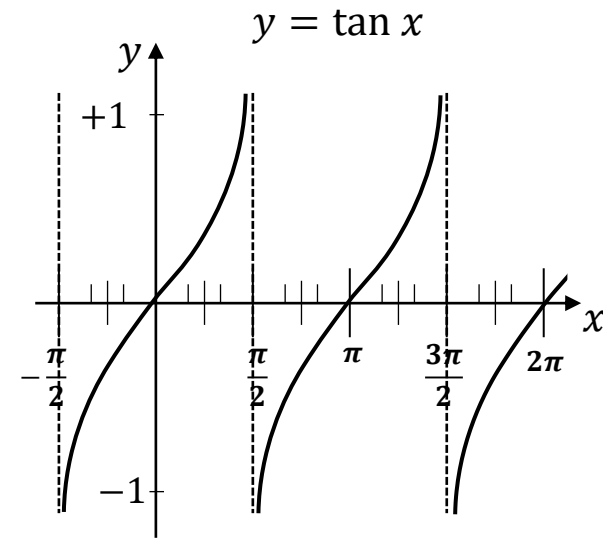
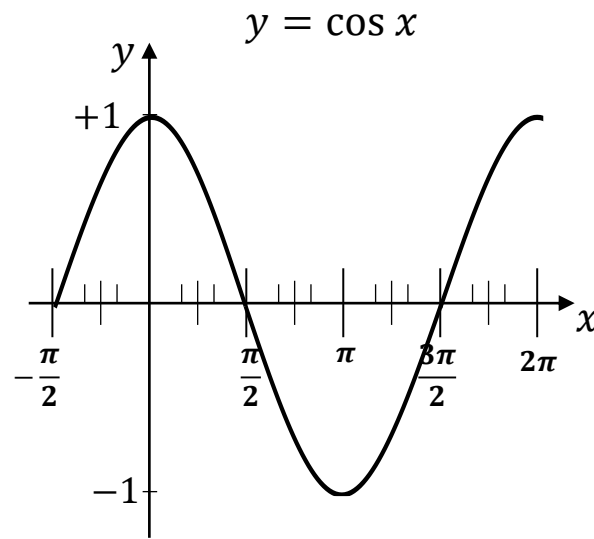
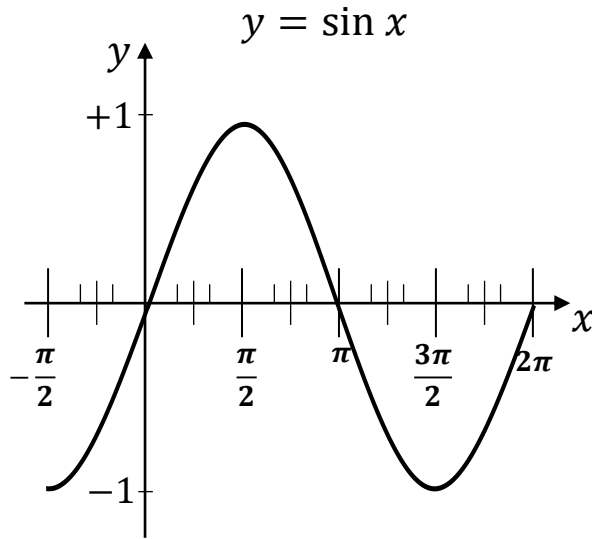
$$y = \cos x$$

$$y = \tan x = \frac{\sin x}{\cos x}$$



x [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$y = \tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	0

三角関数

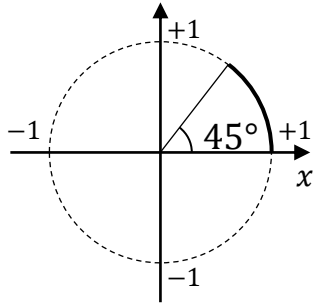


x [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$y = \tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	0

練習問題5

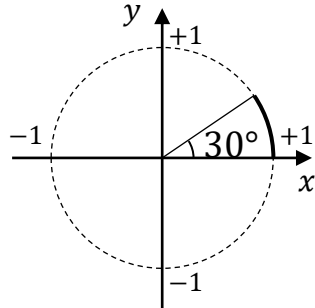
各問に答えよ。各問の円は半径1の単位円である。

(1)



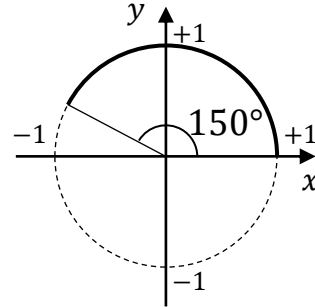
Ans. $\cos 45^\circ =$ _____

(2)



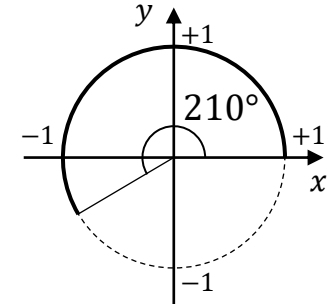
Ans. $\cos 30^\circ =$ _____

(3)



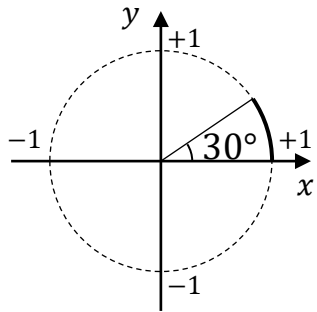
Ans. $\cos 150^\circ =$ _____

(4)



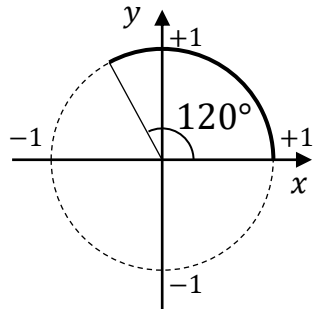
Ans. $\cos 210^\circ =$ _____

(5)



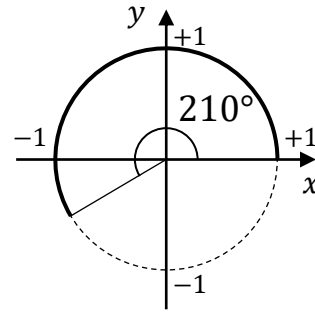
Ans. $\sin 30^\circ =$ _____

(6)



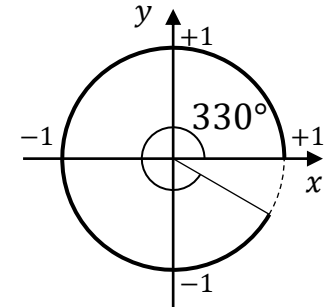
Ans. $\sin 120^\circ =$ _____

(7)



Ans. $\sin 210^\circ =$ _____

(8)

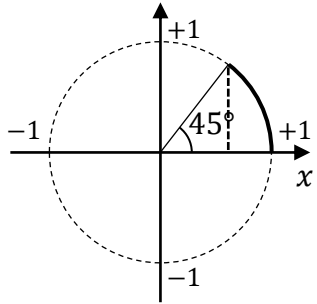


Ans. $\sin 330^\circ =$ _____

練習問題5 (解答)

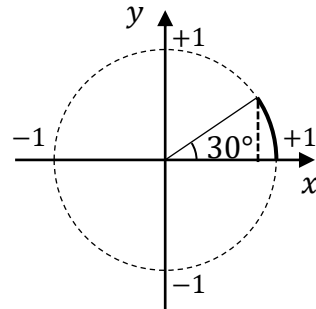
各問に答えよ。各問の円は半径1の単位円である。

(1)



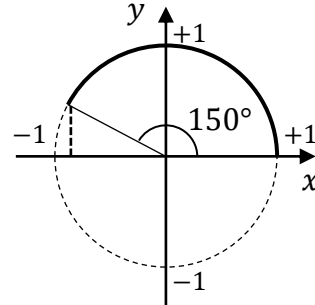
Ans. $\cos 45^\circ = \frac{1}{\sqrt{2}}$

(2)



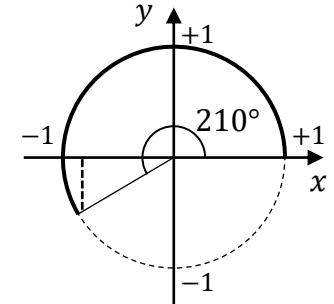
Ans. $\cos 30^\circ = \frac{\sqrt{3}}{2}$

(3)



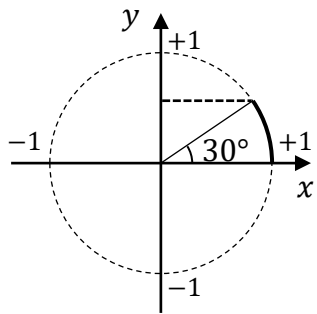
Ans. $\cos 150^\circ = -\frac{\sqrt{3}}{2}$

(4)



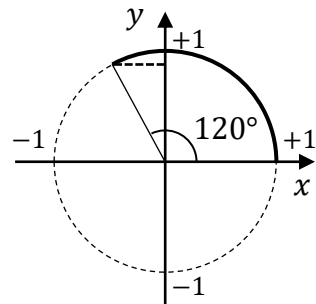
Ans. $\cos 210^\circ = -\frac{\sqrt{3}}{2}$

(5)



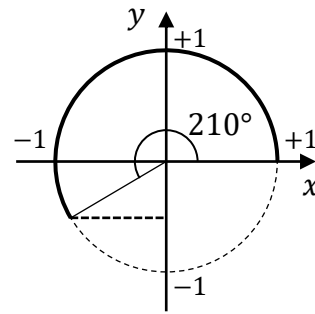
Ans. $\sin 30^\circ = \frac{1}{2}$

(6)



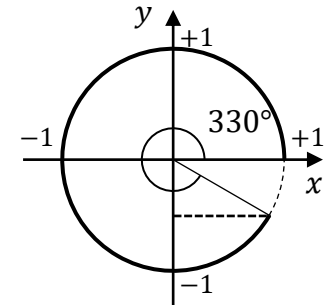
Ans. $\sin 120^\circ = \frac{\sqrt{3}}{2}$

(7)



Ans. $\sin 210^\circ = -\frac{1}{2}$

(8)

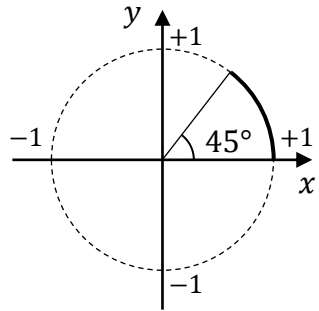


Ans. $\sin 330^\circ = -\frac{1}{2}$

練習問題6

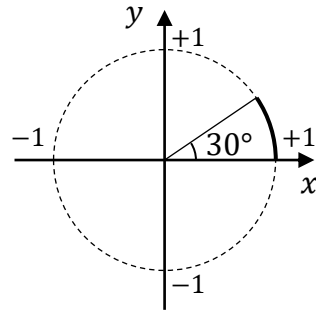
各問に答えよ。各問の円は半径1の単位円である。

(1)



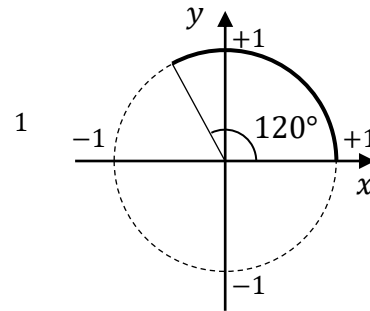
Ans. $\tan 45^\circ =$ _____

(2)



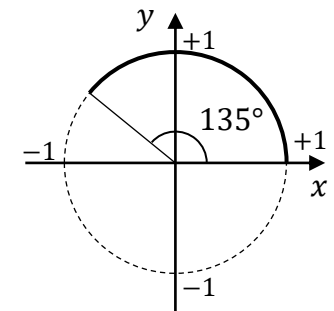
Ans. $\tan 30^\circ =$ _____

(3)



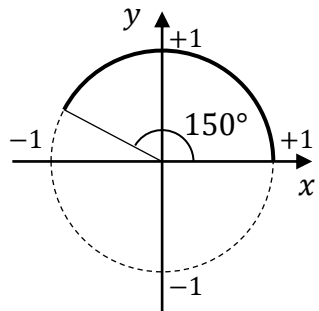
Ans. $\tan 120^\circ =$ _____

(4)



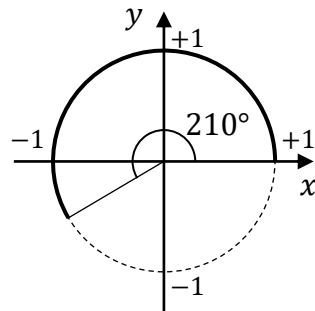
Ans. $\tan 135^\circ =$ _____

(5)



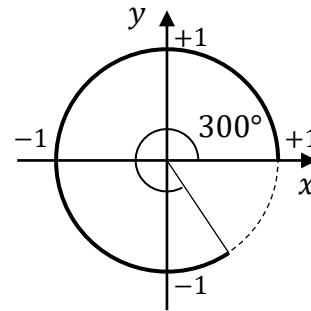
Ans. $\tan 150^\circ =$ _____

(6)



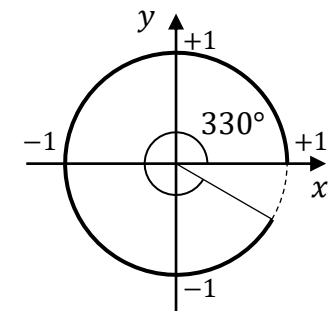
Ans. $\tan 210^\circ =$ _____

(7)



Ans. $\tan 300^\circ =$ _____

(8)

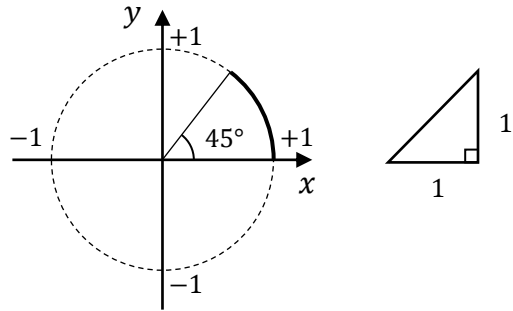


Ans. $\tan 330^\circ =$ _____

練習問題6 (解答)

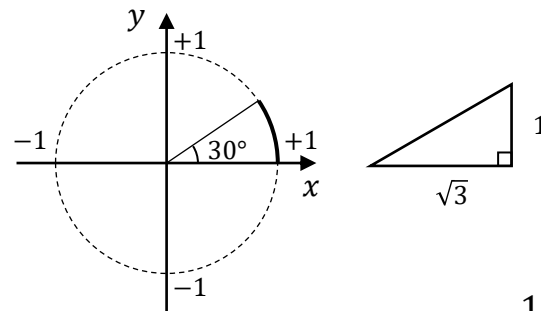
各問に答えよ。各問の円は半径1の単位円である。

(1)



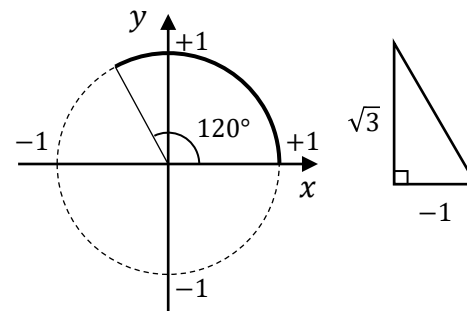
Ans. $\tan 45^\circ = 1$

(2)



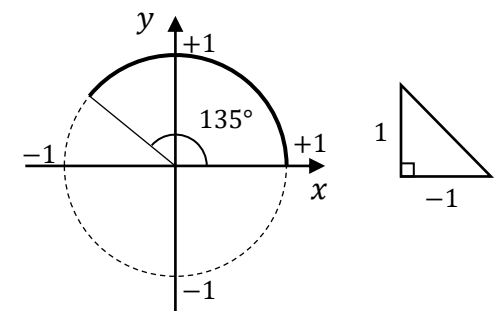
Ans. $\tan 30^\circ = \frac{1}{\sqrt{3}}$

(3)



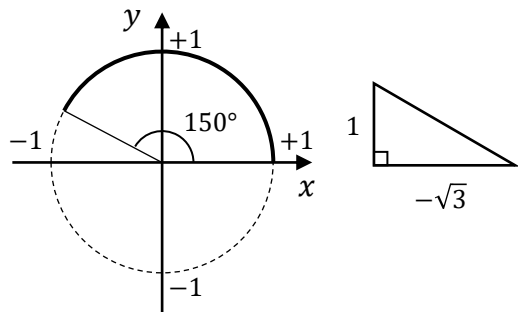
Ans. $\tan 120^\circ = -\sqrt{3}$

(4)



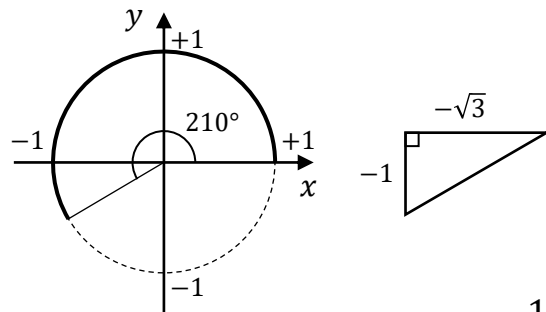
Ans. $\tan 135^\circ = -1$

(5)



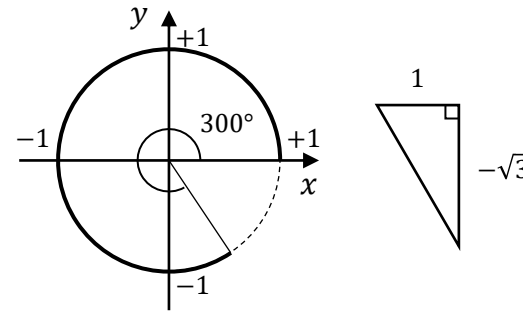
Ans. $\tan 150^\circ = -\frac{1}{\sqrt{3}}$

(6)



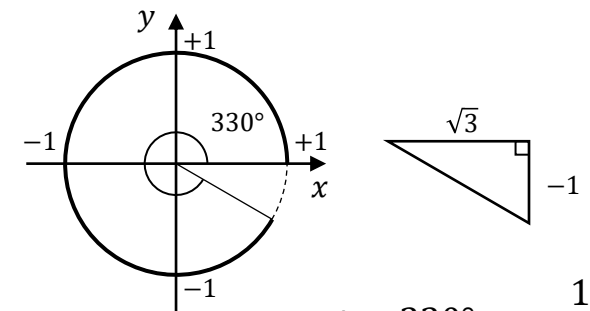
Ans. $\tan 210^\circ = \frac{1}{\sqrt{3}}$

(7)



Ans. $\tan 300^\circ = -\sqrt{3}$

(8)

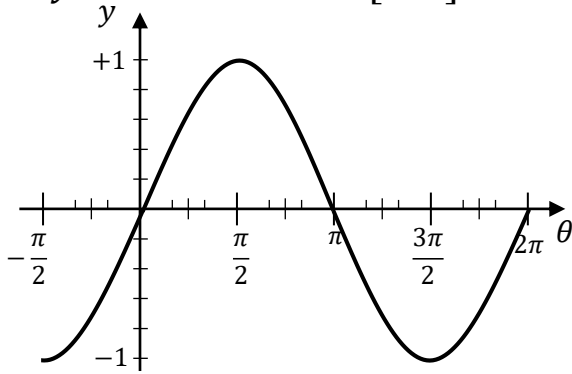


Ans. $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

練習問題7

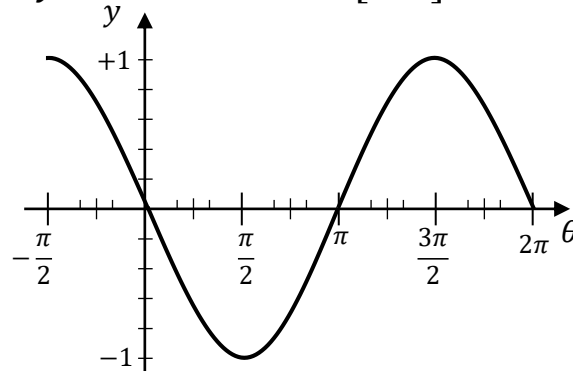
グラフを参照し、各問に答えよ。

(1) $y = 1$ となる角度[rad]示せ



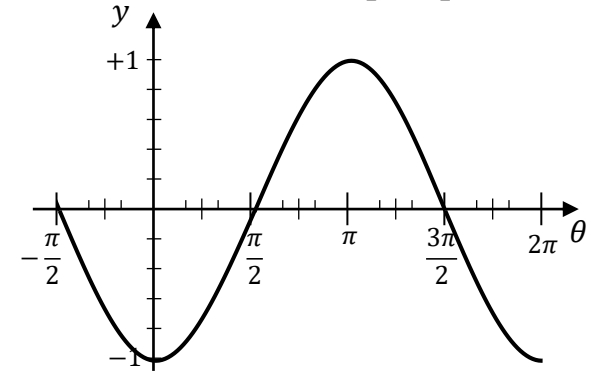
Ans. $\theta =$ _____

(2) $y = 1$ となる角度[rad]示せ



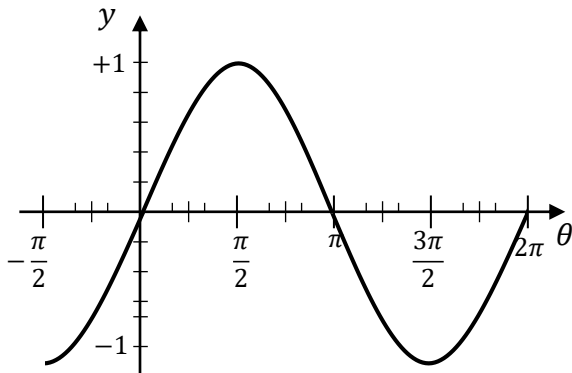
Ans. $\theta =$ _____

(3) $y = 1$ となる角度[rad]示せ



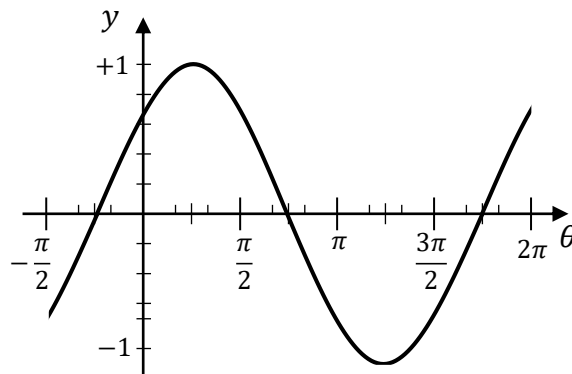
Ans. $\theta =$ _____

(4) $y = 0$ となる角度[rad]示せ



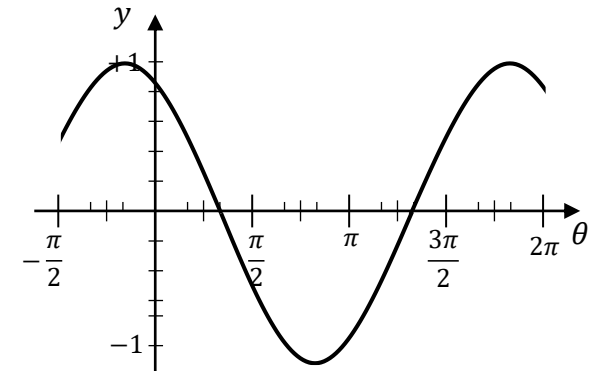
Ans. $\theta =$ _____

(5) $y = 0$ となる角度[rad]示せ



Ans. $\theta =$ _____

(6) $y = 0$ となる角度[rad]示せ

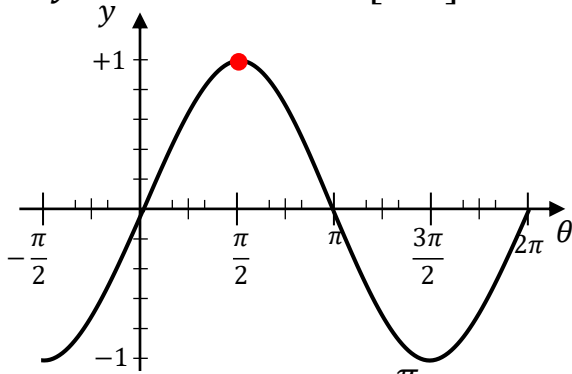


Ans. $\theta =$ _____

練習問題7 (解答)

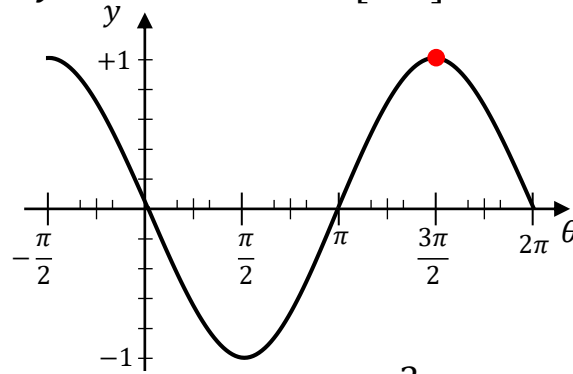
グラフを参照し、各問に答えよ。

(1) $y = 1$ となる角度[rad]示せ



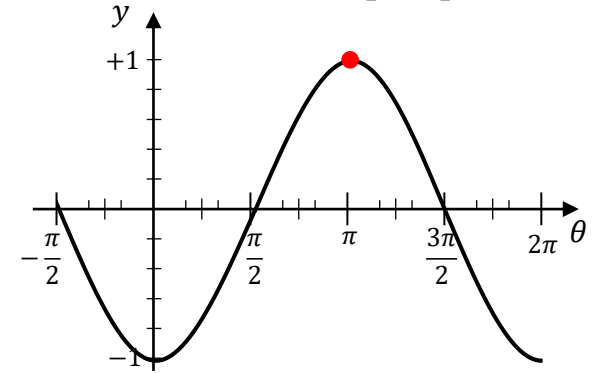
Ans. $\theta = \frac{\pi}{2}$

(2) $y = 1$ となる角度[rad]示せ



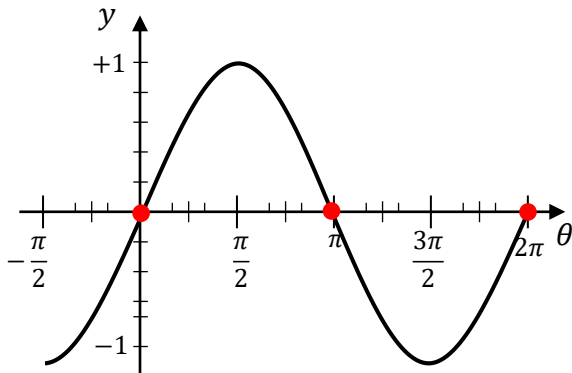
Ans. $\theta = \frac{3\pi}{2}$

(3) $y = 1$ となる角度[rad]示せ



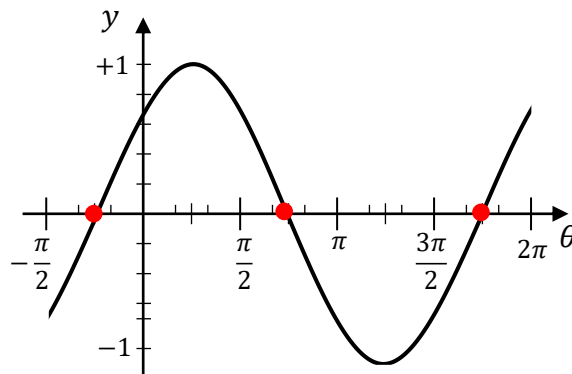
Ans. $\theta = \pi$

(4) $y = 0$ となる角度[rad]示せ



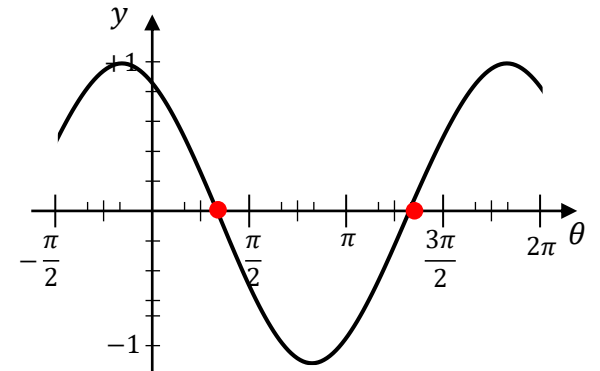
Ans. $\theta = 0, \pi, 2\pi$

(5) $y = 0$ となる角度[rad]示せ



Ans. $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

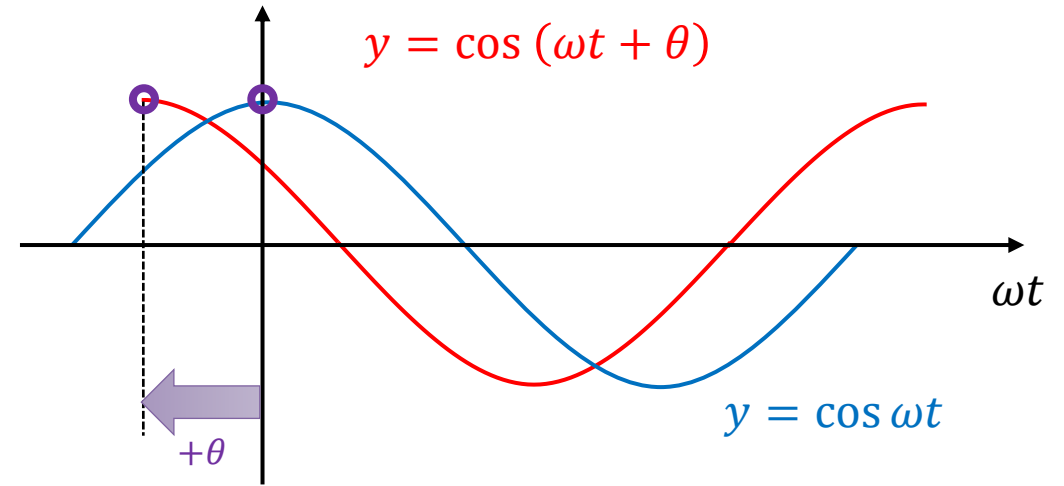
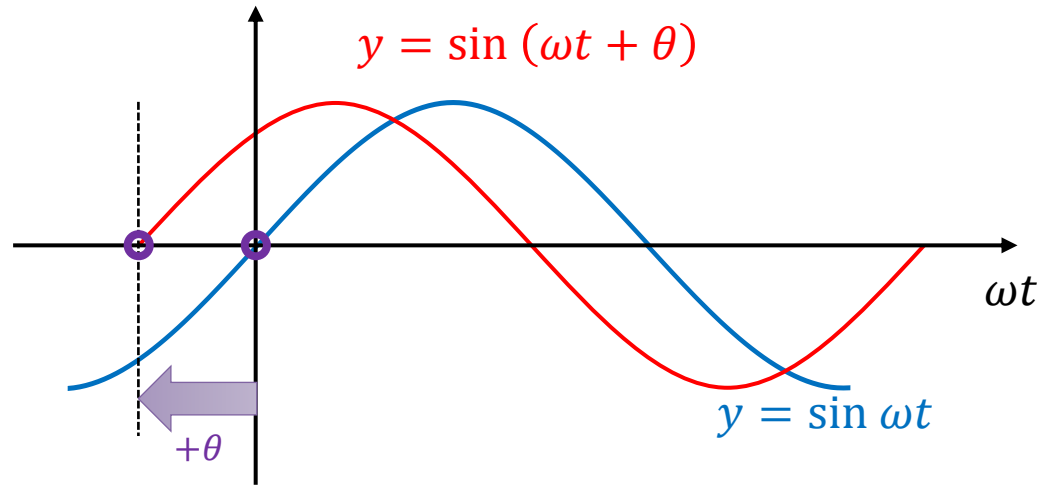
(6) $y = 0$ となる角度[rad]示せ



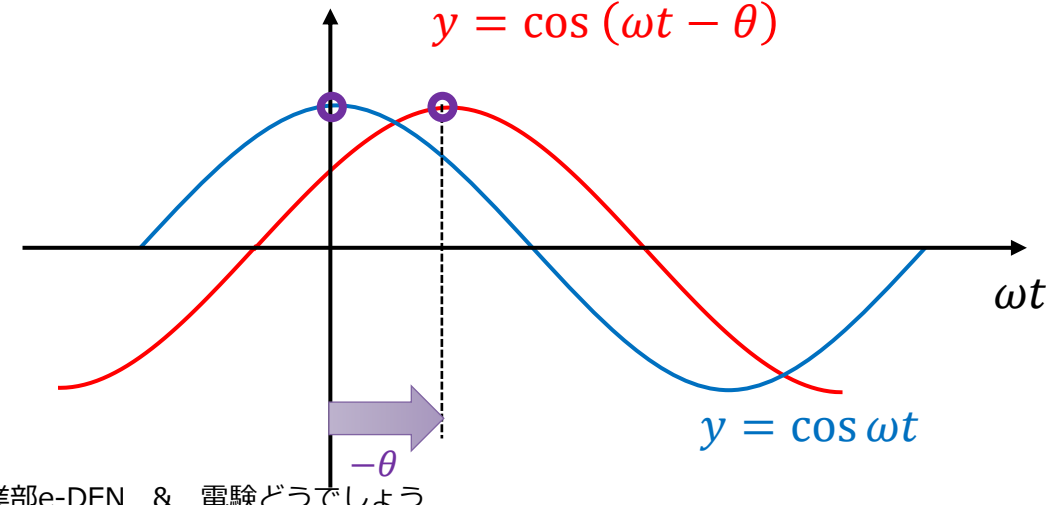
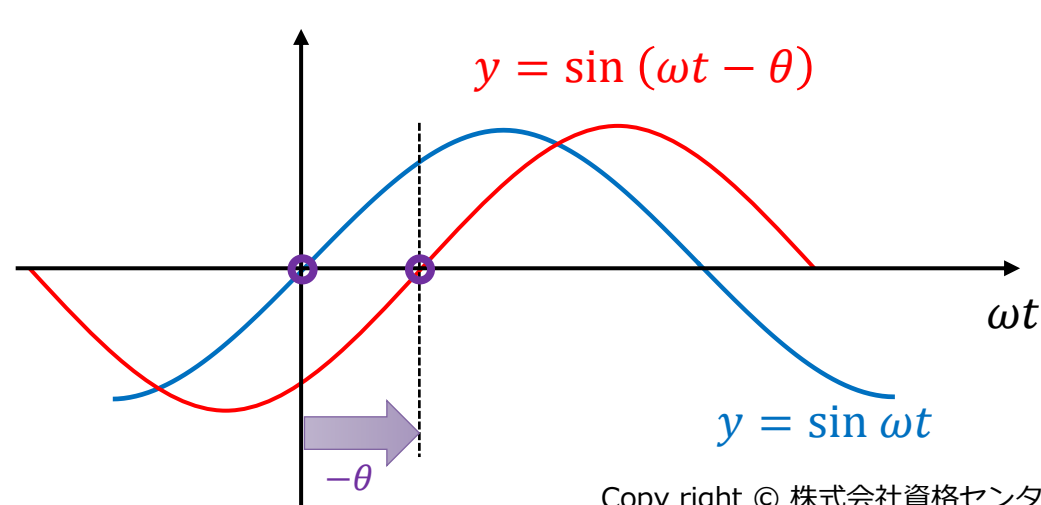
Ans. $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

波形と位相差

位相差が正だと波形は左にずれる



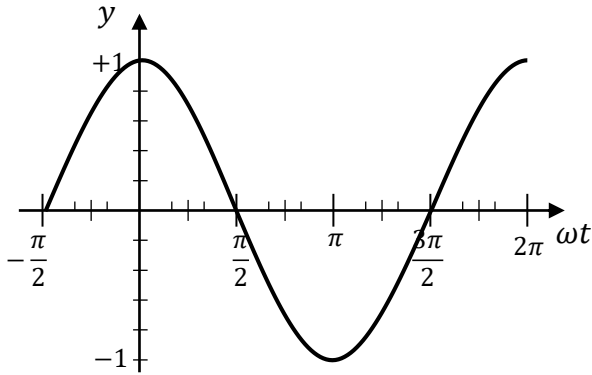
位相差が負だと波形は右にずれる



練習問題8

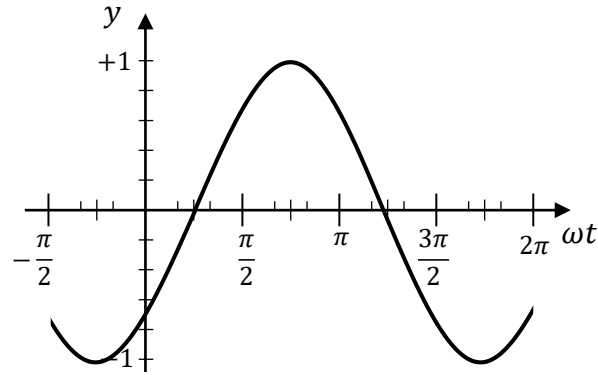
各問の位相差 ϕ の値を示せ。ただし、値は弧度法[rad]で表すこととする。

(1) $y = \sin(\omega t + \phi)$



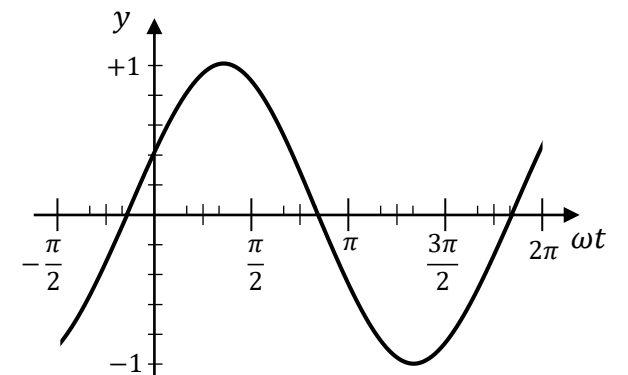
Ans. $\phi =$ _____

(2) $y = \sin(\omega t + \phi)$



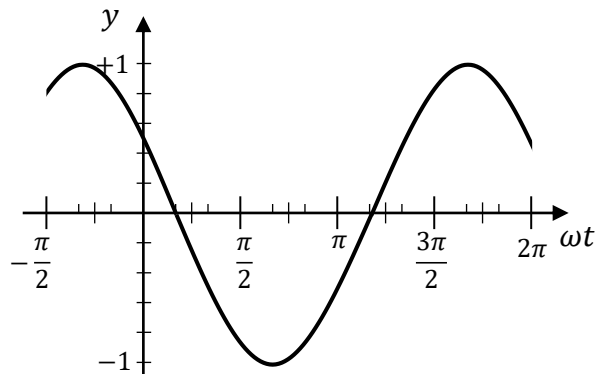
Ans. $\phi =$ _____

(3) $y = \sin(\omega t + \phi)$



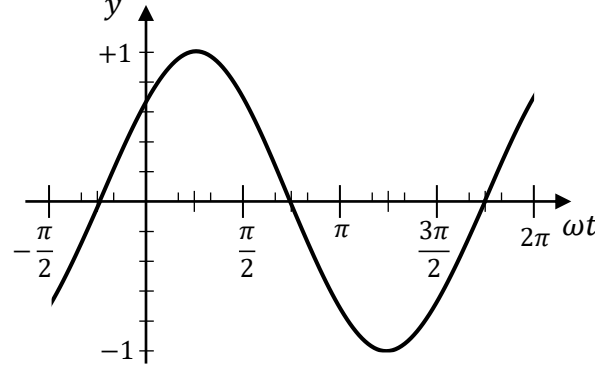
Ans. $\phi =$ _____

(4) $y = \cos(\omega t + \phi)$



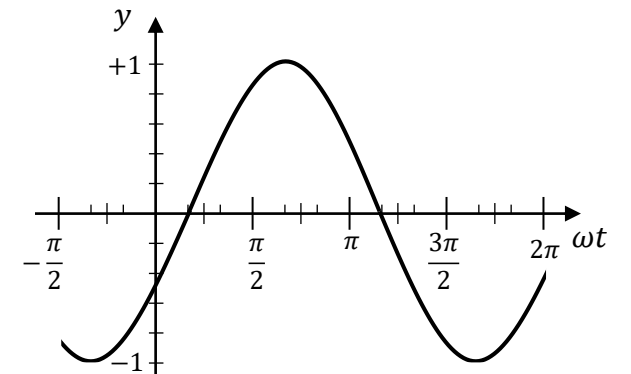
Ans. $\phi =$ _____

(5) $y = \cos(\omega t + \phi)$



Ans. $\phi =$ _____

(6) $y = \cos(\omega t + \phi)$

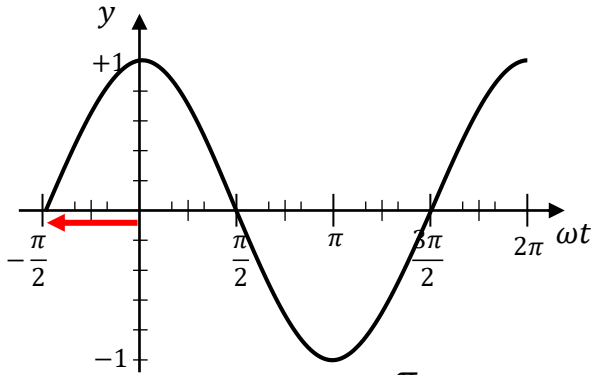


Ans. $\phi =$ _____

練習問題8 (解答)

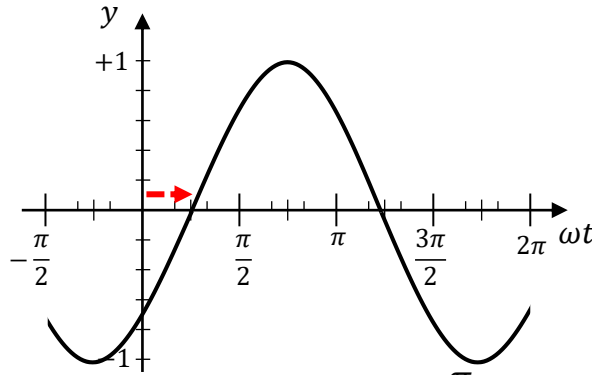
各問の位相差 ϕ の値を示せ。ただし、値は弧度法[rad]で表すこととする。

(1) $y = \sin(\omega t + \phi)$



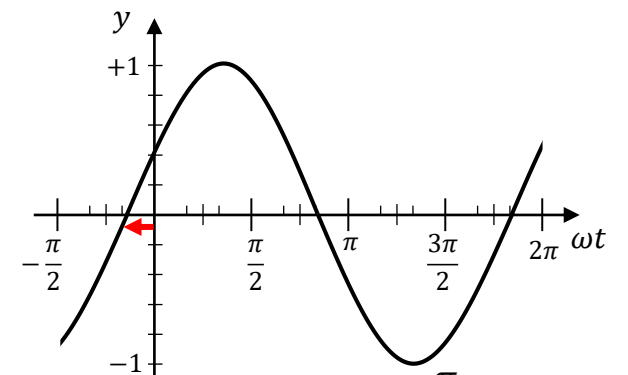
Ans. $\phi = \frac{\pi}{2}$

(2) $y = \sin(\omega t + \phi)$



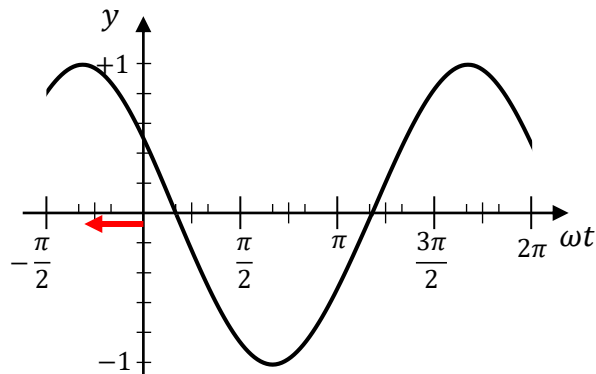
Ans. $\phi = -\frac{\pi}{4}$

(3) $y = \sin(\omega t + \phi)$



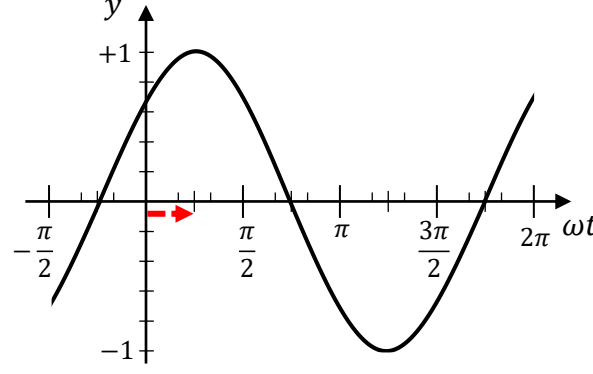
Ans. $\phi = \frac{\pi}{6}$

(4) $y = \cos(\omega t + \phi)$



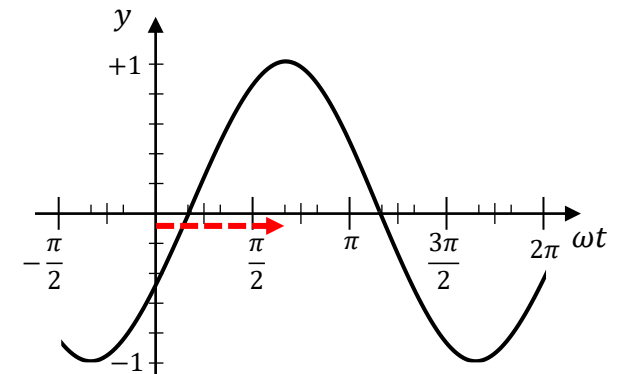
Ans. $\phi = \frac{\pi}{3}$

(5) $y = \cos(\omega t + \phi)$



Ans. $\phi = -\frac{\pi}{4}$

(6) $y = \cos(\omega t + \phi)$

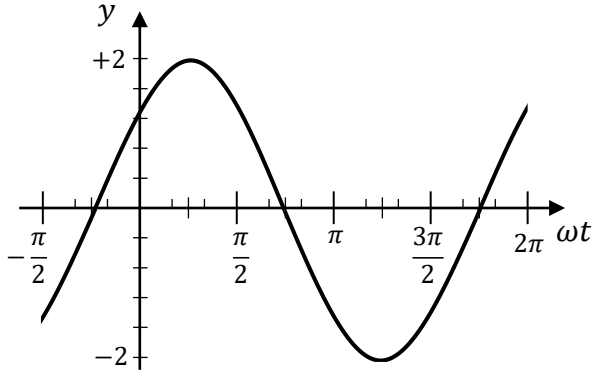


Ans. $\phi = -\frac{2\pi}{3}$

練習問題9

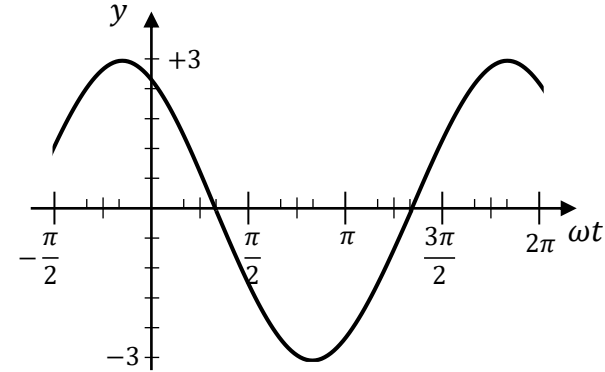
各問の振幅 A と位相差 ϕ の値を示せ。ただし、角度の値は弧度法[rad]で表すこととする。

(1) $y = A\sin(\omega t + \phi)$



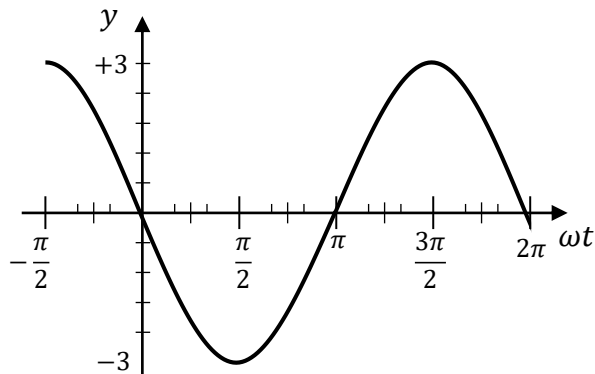
Ans. $A =$ $\phi =$

(2) $y = A\cos(\omega t + \phi)$



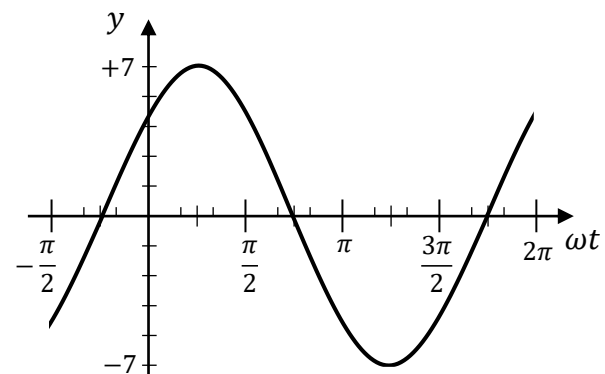
Ans. $A =$ $\phi =$

(3) $y = A\sin(\omega t + \phi)$



Ans. $A =$ $\phi =$

(4) $y = A\cos(\omega t + \phi)$

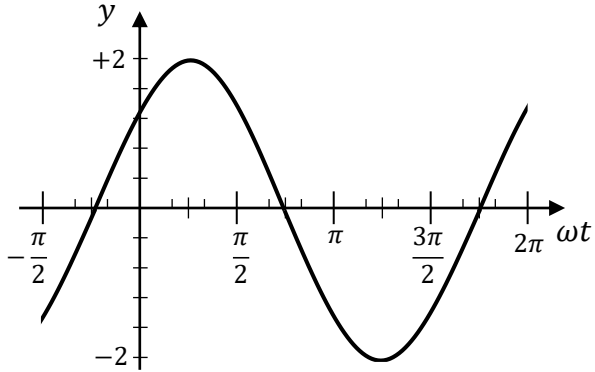


Ans. $A =$ $\phi =$

練習問題9 (解答)

各問の振幅 A と位相差 ϕ の値を示せ。ただし、角度の値は弧度法[rad]で表すこととする。

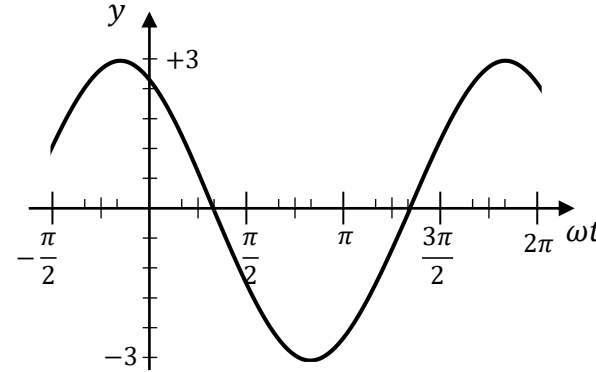
(1) $y = A\sin(\omega t + \phi)$



$$y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$$

Ans. $A = 2$ $\phi = \frac{\pi}{4}$

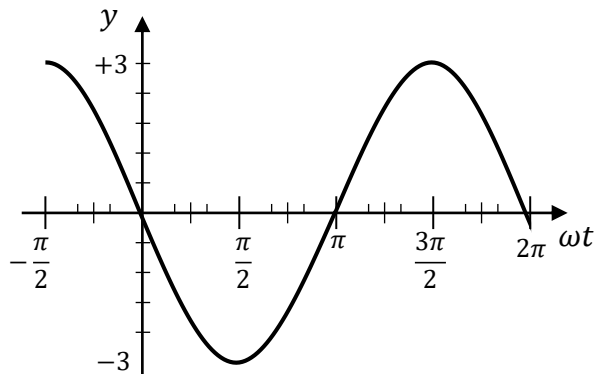
(2) $y = A\cos(\omega t + \phi)$



$$y = 3\cos\left(\omega t + \frac{\pi}{6}\right)$$

Ans. $A = 3$ $\phi = \frac{\pi}{6}$

(3) $y = A\sin(\omega t + \phi)$



$$y = 3\sin(\omega t + \pi)$$

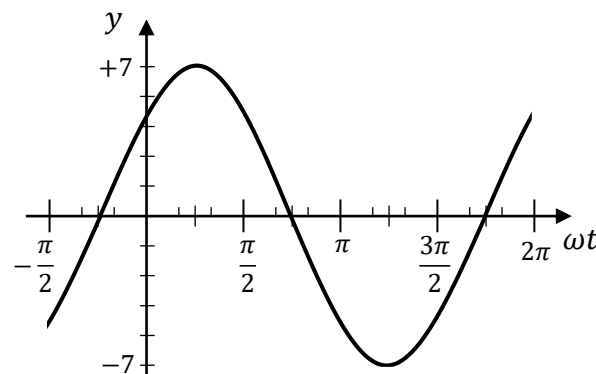
または、

$$y = 3\sin(\omega t - \pi)$$

$$y = -3\sin \omega t$$

Ans. $A = 3$ $\phi = \pi$

(4) $y = A\cos(\omega t + \phi)$



$$y = 7\cos\left(\omega t - \frac{\pi}{4}\right)$$

Ans. $A = 7$ $\phi = -\frac{\pi}{4}$

ご聴講ありがとうございました!!